Space-Time Quantization

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Abstract: It is shown that the postulate of a space-time continuum is not a logical necessity, since it is possible to construct a theory, where the ultimate limit for the smallest measurable distance *a* is finite. This *quantum of length* is a universal constant, like the light velocity c and Planck's constant h. The generalized theory implies that the total energy content of our Universe $E_u = hc/2a$ and that velocities $v > c$ are possible for material bodies, when their energy is approaching E_u. That's surprising, but there are no logical inconsistencies. On the contrary, this theory removes a basic contradiction between relativity and quantum mechanics: the EPR paradox. It accounts also for the mysterious "internal degrees of freedom" of elementary particles, by relating them to possible results of space-time measurements, when the quantum of length *a* is finite.

Introduction

Physics is a science, where one tries to acquire *more knowledge* about the structure and properties of reality at the most profound, accessible level. Occasionally, physics is a more philosophical endeavor, since it is also searching for *deeper insight* in the essence or actual meaning of this knowledge. In classical physics, it was believed, indeed, that physical laws are direct statements about reality, but it appears now that they are statements concerning *the knowledge we can get about reality*. That's very different, since this knowledge results from measurements that are themselves subjected to restrictions.

Albert Einstein initiated this epistemological revolution in 1905, by constructing the theory of *special relativity*. He did this for purely logical reasons, since he had recognized that Maxwell's theory of electromagnetism and Newtonian mechanics were fundamentally incompatible with one another. One theory implied that *the velocity of light in vacuum should have the same value c for all directions and all inertial reference frames,* independently of their relative motions, but this was strictly forbidden by the other theory. Einstein removed this paradox, through a simple, but radical change of the previous concepts of space and time. He considered space and time as being defined by *possible results of measurements.* These measurements are subjected, however, to a very precise and completely unexpected condition. The outcome of any measurement of space and time intervals in inertial reference frames has to be such that when we measure the distance traveled by a light pulse in vacuum during a measured time interval, this has always to yield the same value c.

To understand the importance of this assertion, we have to consider previous ideas about space and time and to explain what is meant by "inertial reference frames". We will also stress the fact that Einstein's concept of physical laws did influence the development of *quantum mechanics.* Actually, we can say that this theory takes into account an other restriction that Nature imposes on certain measurements, and that these restrictions are also related to the existence of a universal constant: *Planck's constant h.*

These facts suggest that Nature could impose a third restriction. Perhaps, we are not aware of it, since the usual assumption yields a sufficiently good *approximation.* Nevertheless, we could be victims of an illusion, when we postulate that this assumption has to be correct under all circumstances. Does there exist a basic assumption that could be false? Yes, present-day theories presuppose that there exists a "space-time continuum" or more precisely, that it should be possible to measure always smaller and smaller intervals of space and time, *without* *any finite limit.* Since this couldn't be experimentally verified, we don't really know if this is true or not, but we have the impression and are conditioned to think that space and time are continuous. We simply *extrapolate* what is known to be true for relatively small intervals of space and time, by assuming that this should also be true for *arbitrarily small* intervals. Actually, we can't be sure that nothing else could occur at some very small scale.

How could we find out? The old algebraic method consists in giving a name to what is unknown and to express its properties by means of an equation. Implicit information can then be made explicit, by purely logical transformations. Let's consider *an ultimate limit for the smallest measurable distance* and call it "a". We will treat it as a yet unknown quantity, but we can postulate that it has to be *a universal constant,* like c and h. The question is then: *is it possible* to construct a theory that takes into account the constants c, h and a, without ending up with logical contradictions when *a* is finite?

The generalized theory has to reduce to the usual one when a = 0, but we want to know if the continuum assumption is a logical necessity or not. The method, which consists in constructing a more general theory and to test its logical acceptability has already been used for *Euclid's postulate about parallel lines.* As soon as an acceptable "non-Euclidean geometry" had been constructed, it was clear that it is not necessary to assume that through a point outside a given line, one can draw *one and only one* parallel line that will never cut the first one. But after that, it took still many years to change the older, deeply rooted ideas.

Psychosocial problems do also arise when one has to abandon the familiar concept of a space-time continuum. The logical possibility and most important physical consequences of *space-time quantization* were established already in the sixties[1]. Since we continued to investigate this problem, other results and ideas were added, but they encountered significant resistance. It may thus be adequate to summarize the results that have been obtained so far and to stress the fact that the underlying principles are very simple and natural extensions of *the general trend of the evolution of physics.*

Changing concepts of space and time

The first intuitive ideas about space and time were based on daily experience, as well in our own childhood as for mankind. We see that the position of a material object can change in the course of time and we realize very soon that this object can't disappear somewhere, to reappear somewhere else. Material bodies have *a continual existence.* We are also able to imagine extremely small particles and we can even reduce material objects in our mind to single points, but we will then assume that such a point-like particle has to move along *a well-defined trajectory*. Moreover, it couldn't move from a given point A to another point B, without passing through a continuous sequence of intermediate points, because of its continual existence. We postulate therefore that the trajectory has to be *a continuous line.* That's the intuitive justification of the concept of a space-time continuum.

Analogies and linguistic tools did also shape our first ideas about space and time. We say, indeed, that objects are moving *in space,* as they do in air or water. This conveys the feeling that space could be some kind of substance. We say that time is *flowing,* since we can measure passing time, by considering the amount of water or sand that is steadily flowing out of a vessel through a small opening. The *clepsydra* was such an instrument, but we can also measure time by means of a *clock* that reproduces always the same unit of time. Thus, we do

[1] A. Meessen: Sur le problème de la quantification de l'espace-temps, Ann. Soc. Sc. Bruxelles, **81**, 254-271 (1967).

represent different instants t by different points on a single time-axis and these points are assumed to form a continuum.

While time has only one dimension, *space* is three-dimensional. This means that it is possible to choose three reference axes, allowing for *independent distance measurements*. The common origin, the directions and orientations of the reference axes can be chosen in a completely arbitrary way, but they should form three different planes. If they were coplanar, they wouldn't allow for independent distance measurements. This results from the fact that we are free to measure the lengths of two sides of a triangle, but the length of the third side can then be calculated. It is already determined and doesn't have to be measured. It is not necessary that the spatial reference axes are perpendicular to one another, but once we have chosen them, as well as a time measuring system, we can say that space and time are *defined* by the possible values of four measurable quantities: x, y, z and t. That's not sufficient, however, to determine the *properties* of space and time. Since they can't be directly perceived, we have to make assumptions and to check their validity.

Aristotle imagined that space is an ensemble of points, forming a continuum, but he postulated that *one of these points has a special status*. It seemed, indeed, that all stars are moving together, as if they were fixed on the surface of a sphere, rotating around an axis that passes through the center of the Earth. This led to the idea that *the center of the Earth coincides with the center of the Universe*. Because of its distinguished position, this point would then have no reason to move in whatever direction. The center of Earth should thus be absolutely *motionless*. The idea that the center of the Earth occupies a privileged position in space seemed to be confirmed by the fact that heavy bodies are freely falling along *vertical* lines. All these lines are passing through the center of the Earth.

Aristotelian physics was apparently based on good observations and a secure induction, but Nicolas de Cusa and Giordano Bruno questioned the concept of a limited Universe, while Copernicus and Kepler centered the Universe on the Sun. This was already quite revolutionary, but Galileo recognized that a much more profound revision of Aristotelian physics was actually required. This theory implied, indeed, that any object that is motionless with respect to the center of the Earth is in a state of *absolute rest*. Motion seemed thus to be comparable to life: a being is living or not. We are still saying that objects can be "animated" by motions, and a terrestrial object tends to loose its motion. That means that the velocity $v = 0$ with respect to the center of the Earth seemed to have a special status. Galileo saw that this couldn't be true, since velocities can be added to one another by changing the reference frame. *All velocities are relative*. They depend on the chosen reference frame.

Galileo formulated even the principle of relativity and the principle of inertia, but since he tried always to justify his propositions in terms of experimental observations, he considered only what happens for bodies near the surface of the Earth. The *principle of relativity* states that a physical law is a relation that is independent of the chosen reference frame, while the *principle of inertia* requires that the velocity of a material body remains constant (whether it is zero or not) as long as this body is not perturbed by some applied force.

Newton adopted a broader view, encompassing heavenly motions as well as terrestrial ones. The principle of inertia required then that the acceleration is zero, when the applied force is zero, but this can only be true for a particular set of reference frames, the so-called *"inertial reference frames"*. They are not accelerated relative to one another, and they are physically distinguished with respect to other frames, since they are the only ones where the acceleration of any material body is zero *when the applied force is zero*. Here, we consider "true forces". They do not only appear as causes of acceleration, but do also result from real physical interactions with other bodies. The existence of such a privileged set of reference frame was very astonishing, since the principle of inertia should be valid for all material bodies and all types of forces, always and everywhere in the whole Universe.

Newton tried to justify this, by postulating that there exists *everywhere* a point that is in a state of absolute rest. The ensemble of these points would constitute *"absolute space"*. Inertial reference frames were then those frames that are not accelerated with respect to absolute space. To define the acceleration, it seemed necessary to assume also the existence of *"absolute time"*, flowing always and everywhere in exactly the same way.

Some physicists thought that absolute space could be materialized by the *ether*, which had been considered already during Greek Antiquity, since it seemed difficult to accept the existence of "nothingness", even at places where there is no matter at all. Maxwell used this concept to develop a theory that accounted for all known facts concerning electricity and magnetism, by assuming that electric and magnetic fields correspond to modifications of this ether. This led to logically consistent equations, predicting the existence of electromagnetic waves. They would always propagate in vacuum (thought to be pure ether) at a well-defined velocity c. Its value was determined by electrical and magnetic measurements, but was also equal to the measured value of the velocity of light in vacuum. Light could thus be considered as a particular type of electromagnetic waves.

That was a beautiful theory, but Einstein saw that it led to a fundamental problem. The principle of relativity would require that *the velocity c is a universal constant* for all inertial reference frames, while Newton's theory did not allow for such a velocity. Einstein showed that this problem could be solved, by abandoning the concept of absolute space and time. We shouldn't consider space and time any more as some kind of physical reality, but *define space and time by possible results of measurements.* They allow us to specify the space-time coordinates (x, y, z, t) of any given event in a given reference frame. When these measurements are performed in another reference frame for the same event, we get different results: (x', y', z', t'). Einstein *postulated,* that for inertial reference frames, these results are related to one another in such a way that a measurement of the velocity of light in vacuum yields always the same value *c*.

This idea changed physics in a very profound way, since it indicated for the fist time that *physical laws should only be considered as statements concerning the knowledge we can get about reality.* As we mentioned in the introduction, quantum mechanics did confirm this rule, but even today, this is not always emphasized.

Let's recall that Planck had shown (in 1900) that certain properties of electromagnetic radiation inside a hot cavity could be explained, by assuming that *the energy of a light wave* can only be increased or decreased by finite quantities. They are equal to $h\nu$, for waves of given frequency ν, while *h* is now called "Planck's constant". Einstein was much bolder, since he proposed (in 1905) to consider that *any light-wave is equivalent to an ensemble of particles,* each one of them carrying an energy E and a momentum p, defined by

$$E = h\nu \qquad \text{and} \qquad p = h/\lambda \tag{1}$$

where ν is the frequency and λ the wavelength of the light-wave. This was partly an application of the theory of relativity, since this theory predicted that for a given inertial frame, the energy E and momentum p of a freely moving particle should be such that

$$(E/c)^2 = p^2 + (m_o c)^2 \tag{2}$$

The momentum p = mv, where v is the velocity of the particle, while m has the dimension of a mass. When the particle is at rest (p = 0), the energy $E = m_o c^2$. That a material particle contains a certain amount of energy, because of its *rest mass* m_o, was completely unknown in classical mechanics. This shows already that the restrictions that Nature imposes on our measurements can actually lead to *more detailed knowledge.* Moreover, we end up with a greater unity of physics, since it follows from (2) that $E/c = p$, when $m_o = 0$. This is compatible with (1), since for electromagnetic waves in vacuum, $\lambda = cT$, where the period of oscillation $T = 1/\nu$. Thus, light is composed of particles and *all particles obey Einstein's relation (2).*

Niels Bohr tried (in 1913) to understand atomic spectra. He assumed that electrons are moving inside an atom along *well-defined trajectories* and that only some of these trajectories are allowed, to account for "quantization rules" that could be deduced from empirical facts. These rules involved Planck's constant h. Although they could be justified by known phenomenology, they were very mysterious. It was thus quite remarkable that Louis de Broglie could justify their existence (in 1924), by assuming that a material particle is always accompanied by an "associated wave". Its properties should be defined by (1). This postulate did also increase the unity of physics, since the relations (1) would not only apply to photons, but also to material particles. Erwin Schrödinger established then (in 1926) a wave equation and solved it for bound particles, while Max Born treated (also in 1926) the problem of scattering, where a collision can send a given particle in many different directions. This led to the *probabilistic interpretation* of the quantum-mechanical wave function.

In the meantime, Werner Heisenberg had already formulated quantum-mechanical laws by following a completely different road. He questioned the concept of well-defined trajectories for atomic electrons, since they presuppose that it should be possible to get *an absolutely precise knowledge* about the position and velocity of an electron at any particular instant. According to his understanding of Einstein's theory of relativity, physical laws should only express relations between *possible results of measurement.* The positions and velocities of atomic electrons should thus not be defined by extrapolating daily experience, but in terms of possible spectroscopic observations. Heisenberg expressed this knowledge by means of an ensemble of numbers and described (already in 1925) the motions of atomic electrons by means of a "matrix mechanics" that was more abstract, but equivalent to "wave mechanics".

Heisenberg formulated then (in 1927) his famous "uncertainty relations". Actually, he followed up his initial idea, but expressed it now in terms of the Einstein - de Broglie relations (1). They imply that E and p are only sharply defined, when the wave function oscillates *always* at the same frequency ν and *everywhere* with the same wavelength λ. This is usually not true, since the wave is only oscillating within a limited region of space and time, but the values of p and E are then not known with absolute precision. The uncertainties are $\Delta p > h/\Delta x$ and $\Delta E > h/\Delta t$, where Δx and Δt are the allowed uncertainties in space and time.

Heisenberg took great care to present these theoretical relations, as consequences of the process of measurement. For instance, when one knows exactly the direction of motion of a particle and when one determines then its position, by means of a slit of width Δx, one looses the previous knowledge, because of diffraction processes. We will end up with an uncertainty $\Delta p > h/\Delta x$ in the direction of the x-axis, although before this measurement, Δp was zero. This fact has often been interpreted - too narrowly - as meaning that *the observing subject* perturbs the object he wants to study. It is more appropriate to say that "Nature imposes restrictions on our measurements" and that they are related to the existence of universal constants. In this case, it is Planck's constant h. It has the same value for all directions and all inertial reference frames, like the light velocity c.

This parallelism is of fundamental importance, but it wasn't even obvious for Einstein. In a private discussion with Heisenberg (that took place already in 1924) he said[2]: *"You are talking about what we know about Nature and not about what Nature is really doing".* Why was Einstein so reticent? As a student, he had been impressed by Ernst Mach's criticism of classical mechanics, but later, he rejected the extreme and sometimes blind positivistic

[2] W. Heisenberg : Der Teil und das Ganze, Piper & Co,1963, dtv, 1973, p. 85

philosophy[3]. Heisenberg's intuition was fruitful and basically correct. We will thus go one step further in the same direction.

The basic postulate of space-time quantization

When there exists a finite limit for the smallest measurable distance, it is meaningless to formulate a physical law that would tell us what happens at a smaller scale, since *physical laws should be verifiable, at least in principle.* Our usual theories imply, however, that physical laws can be expressed by means of differential equations, assumed to be valid for *infinitely small* intervals of space and time. Perhaps, they are only approximations that cease to be valid at some extremely small scale. The laws of classical mechanics were very good for a certain domain, but they had to be replaced by those of relativistic mechanics and quantum mechanics for larger domains.

To set up the more general laws that take into account the existence of a finite limit *a* for the smallest measurable distance, we have to proceed very carefully, since we enter in unknown territory. What can be safely assumed? Past experience tells us that *a has to be a universal constant, like c and h.* Distance measurements could then be performed, in principle, by successive juxtapositions of the same smallest measurable length. This yields the basic postulate of the generalized theory:

An ideally exact distance measurement along any direction
in any inertial reference frame can only yield integer multiples
of the same universally constant quantum of length a.

Let us chose an x-axis that has a given direction, orientation and origin. When we measure the coordinate x along this direction, by starting at the origin x = 0, as we normally do, the only *possible* values of x are x = 0, ±a, ±2a, ±3a, … *The coordinate x is quantized,* so that the smallest possible difference between two possible values of x is equal to *a.* Usually, we don't perform ideally exact measurements. There remains then some uncertainty about the possible values. This uncertainty can be determined, as well as the average (or most probable) value. Even when x is quantized, *the average value is not quantized.* It results indeed from an explicit or implicit calculation, based on the probability distribution for different possible results. Usually, we are talking about the average position of a particle, and this is a continuous variable, even when space is quantized.

When we perform *ideally precise measurements* of all (x,y,z,ct) space-time coordinates by starting at the origin of the chosen inertial reference frame, we can only get integer multiples of *a* for everyone of these coordinates. This yields a *"space-time lattice"* that depends on the chosen origin and the chosen directions of the reference axes, but the lattice constant *a* is always the same.

To find out if the laws of relativistic quantum mechanics can be generalized, to account for c, h and a, we have to recall what is essential in the usual continuum theory. Einstein showed that the laws of classical mechanics have to be modified in such a way that the energy E and momentum p of a *freely* moving particle are related to one another by means of (2). Since a free particle is unperturbed by external forces, neither its direction of motion, nor the values of p and E can change in the course of time. We can thus choose the direction of motion of the particle as the direction of the x-axis. The *possible positions* of the particle will then be

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[3] P. Frank: Einstein, Mach, and logical positivism, in Albert Einstein: Philosopher-Scientist, P.A. Schilpp (editor), Vol. I, 1949. p. 271-286.

determined by the corresponding value of x. This applies as well to an idealized point-particle as to the center of mass (or center of gravity) of an extended body.

In classical or relativistic mechanics, we describe the motion of this particle, by assuming that it has a well-defined position at any particular instant t, *whether we know this position or not.* The rate of variation of this position during a very small time interval (dx/dt) defines then the instantaneous velocity v. The quantum-mechanical description of motions is radically different, since we consider the propagation of a "wave", but this is a mental construction, synthesizing all the knowledge we can get about the particle. To be more precise, we will say that *to any particular point P in space and time* (where the particle could be found in the chosen reference frame), *we associate a* y*-vector* that is contained in a complex plane. In shorthand notation: $\psi = Ae^{i\varphi}$. This means simply that A is the amplitude of the ψ-vector, as indicated in figure 1, while φ is an angle that defines its orientation with respect to the so-called imaginary axis. Usually, we assume that x and ct can have any values, but the same concepts remain valid when x and ct are quantized. Instead of a space-time continuum, we get then a *space-time lattice* for the chosen reference frame.

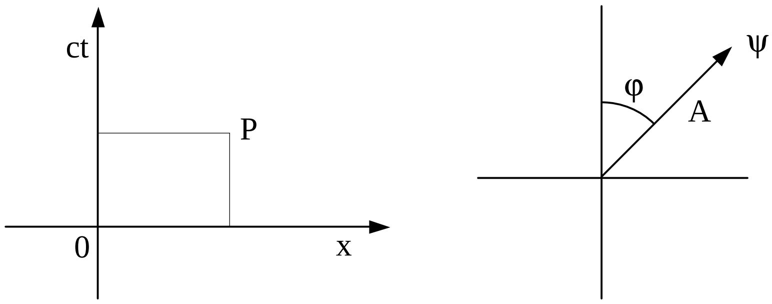

Figure 1. In quantum mechanics, we define the knowledge we can get about a particle, at any point P of space and time where it could be localized, by a ψ-vector in an abstract plane

The square of A yields *the probability to find the particle at the point P.* Since A can vary for different points P, we get in general, a probability distribution in space and time. The angle φ can also vary, but the rate of variation of φ yields information about the *state of motion* of the particle. When the particle has a sharply defined energy E and a sharply defined momentum p, the ψ-vector has always and everywhere the same amplitude A, but it rotates with a well-defined angular velocity along the x-axis and the ct-axis. Mathematically, it is sufficient to say that $\varphi = k.x-\omega.t$, where $k = 2\pi/\lambda$ and $\omega = 2\pi\nu$. The older relations (1) acquire now a new meaning. Actually, they *define new measurable properties.*

In the usual theory, the coordinate x is a continuous variable. Since ψ contains the factor e^{ikx}, the first and second order partial derivatives with respect to x are given by

$$\partial_x \psi = ik\psi \qquad \text{and} \qquad \partial_x^2 \psi = -k^2\psi \tag{3}$$

The first expression defines the *local slope* (or rate of variation) of the ψ function along the x-axis. It is equal to Δψ/Δx, where Δψ is the change of ψ when x is increased by Δx, where Δx is very small. The second expression defines the local *curvature* (or rate of variation of the slope). The minus sign arises from the fact that $i^2 = -1$. From a mathematical point of view, it is assumed that we can go over to the limit where the interval Δx becomes *infinitely small* (can be reduced more and more, without ever reaching any finite limit). But from a *physical* point of view, it is sufficient that the interval Δx is small enough, to insure that the result doesn't change when the size of the interval Δx undergoes further reductions.

When there exists a finite quantum of length *a*, we have to replace the partial derivatives (3) by *finite differences*, but before we do that, we have to call attention on a very important fact, usually buried under a layer of formalistic jargon. First of all, we recall that the relations (3) were obtained by considering a ψ functions that contains the factor e^{ikx}, where k is constant. This means that the momentum $p = h/\lambda = \hbar k$ is *sharply defined* (with $\hbar = h/2\pi$). There is no uncertainty at all, but this is not always true. In general, ψ does not vary everywhere with some particular wavelength λ, but we can use the same relations (3) to define *"local values"* of the momentum p. When the local values are different at different places, there will exist some uncertainty about the actual value of p.

The definition of p, which is used in *classical mechanics*, corresponds only to the average value of the quantum-mechanically defined values of p. Quantum mechanics yields thus a much more detailed description than the classical one. Actually, we can deduce everything we want to know about the possible positions x and the possible momenta p of the particle, by inspecting its ψ function. This is also true for the time variable t and the energy $E = \hbar\omega$, since

$$\partial_t \psi = -i\omega\psi$$

The time t can be measured, in principle, by considering the distance *ct* that a light pulse would travel during the time t. The foregoing expression can thus be rewritten by considering the partial derivative with respect to ct, but ω has then to be divided by c. Now, comes the essential step to get a physical law. *In relativistic quantum mechanics, we postulate that Einstein's relation (2) is still valid for the local values of E and p.* It follows that the ψ function for a free particle in a given inertial reference frame has to satisfy the differential equation

$$\partial_x^2 \psi - \partial_{ct}^2 \psi = K^2 \psi \tag{4}$$

where $K = m_o c/\hbar$. This is the famous "Gordon-Klein equation". It accounts for the existence of two universal constants, c and h. We have to generalize this equation, when the smallest measurable distance is finite. To do that, we will define the local value of k and p *at the smallest possible scale*, by means the following expression:

$$D_x^2 \psi = \frac{\psi(x+a) + \psi(x-a) - 2\psi(x)}{a^2} = -\frac{\sin^2(ka/2)}{(a/2)^2}\psi(x) \tag{5}$$

This defines the curvature at the point x and is a generalization of the second order partial derivative (3) when *a* is finite. The last expression in (5) is obtained when ψ varies like e^{ikx}, but we can use this expression for any function ψ, to define the local value of k. When we replace the partial derivatives in (4) by corresponding finite differences, we get instead of Einstein's relation (2), the *generalized energy-momentum relation*

$$\sin^2(\pi aE/hc) = \sin^2(\pi ap/h) + \sin^2(\pi a m_o c/h) \tag{6}$$

This is a new physical law. It tells us how the energy E depends on the momentum p for any free particle of given rest-mass m_o in any (arbitrarily chosen) inertial reference frame, whether $a = 0$ or not. The last term is a constant, depending on the rest mass m_o. It is expressed here in such a way that we get always $E = m_oc^2$ when p = 0. Einstein's relation (2) is a particular case (where a = 0) or an approximation that is even valid when space and time are quantized, as long as the arguments of the sine functions in (6) are sufficiently small. This will be true for relatively small values of E, p and m_oc. For large values, there will be differences.

Surprising, but logically consistent results

We left the value of the quantum of length *a* completely undetermined, since we wanted to *avoid any arbitrariness.* It is possible to get a universally constant length, by combining c and h with the gravitational constant G, for instance. This yields the Planck length (10^{-35} m). It can be expected to play an important role in "quantum gravity" and thus for the physics of black holes[4], but we are looking for general physical laws that are valid *everywhere* in the whole universe, even when there is no gravitational field at all. The generalized energy-momentum relation (6) does actually apply to *free particles.*

The prominent feature of this relation is that the sine functions lead to a periodic structure of the energy-momentum space. The possible values of p that give rise to *different states of motion* are thus contained in a domain that is centered on p = 0, but is limited by ±h/2a. Since this is also true for E/c, there exists an upper limit for the highest possible energy. This is *the total energy content of our Universe:*

$$E_u = hc/2a$$

No particle could ever have a higher energy, when this energy is finite[1]. It follows that there is also a limit for the highest possible rest energy $E_o = m_oc^2$. It is reached when the last term of (6) is equal to 1, but for real (measurable) values of E and p, there exists then only one possible state of motion: absolute rest (p = 0), while $E = E_u$. This provides *a new definition for inertial reference frames.* As Newton suspected it is related to something that is in a state of absolute rest, but this is not "absolute space". We can say that if a material body would have a rest energy that is equal to the total energy content of our Universe, there would be no energy left that could appear in the form of kinetic energy. This body would thus have to be in a state of absolute rest. The continuum assumption (a = 0) and Einstein's relation (2) are equivalent to the (arbitrary) postulate that the total energy content of our Universe should be infinite. Inertial reference frames remain then intrinsically undefined.

For photons ($m_o = 0$), the generalized energy-momentum relation (6) allows for E/c = p even if their energy E would be very close to E_u. For material particles, there are differences with respect to the usual continuum theory when their energy E is approaching E_u. This can be expressed most clearly by considering their velocity v. Let's recall that in quantum mechanics, a particle is localized in space and time to a certain extend, when the associated wave function vanishes outside a limited domain. This can be viewed as resulting from an interference effect, when one considers a superposition of waves that are characterized by different, but nearly identical values of k. The *group velocity* of such a "wave-packet" defines the classical velocity or quantum-mechanical average velocity. Its value $v = d\omega/dk = dE/dp$. The generalized energy-momentum relation (6) leads now to the astonishing conclusion that the velocity v *can be larger than c*, when the energy of a material particle is approaching the value of E_u. Actually, it is necessary that $E > E_u/2$.

[4] L. Smolin, Atoms of space and time, Scient. Amer. **290**, Jan. 56-65 (2004).

This effect is unimportant for any practical purposes (like space-travel, for instance), but it is essential as a matter of principles, since *the familiar statement that velocities $v > c$ are forbidden is only true when $a = 0$.* We can even show[5] that it is necessary that a is not zero, to insure the internal coherence of physics. To stress the importance of this fact, we recall that Einstein had discovered that Maxwell's theory of electromagnetism and Newtonian mechanics are not really compatible with one another. Einstein created the theory of relativity to restore harmony, but he discovered also that quantum mechanics and relativity don't really fit together.

Quantum mechanics implies, indeed, that we can consider a system that is composed of two particles, so that a measurement performed on one particle will *instantly* provide information about the other particle, however far it may be from the first one. This should be impossible, according to relativity. Einstein concluded therefore that quantum mechanics does not provide a *complete* description of reality. This statement has often been interpreted as unwillingness or incapacity to accept quantum-mechanical principles, and in particular its probabilistic interpretation, but Einstein had the courage to repeat until his death that he did not consider quantum mechanics as the ultimate truth. Space-time quantization yields a more general theoretical framework and resolves the EPR paradox[6].

This paradox results from the conviction that one measurement can only influence the outcome of another measurement, far away, when there exists some (causal) physical interaction. Today, we attribute such an interaction to an exchange of *virtual particles,* existing only during very short time intervals. Because of Heisenberg's uncertainty relations, they can thus have extremely high energies, and this is now compatible with superluminal velocities. They are not strictly forbidden anymore, when a is finite. Space-time quantization is thus not only logically possible, but *removes even a logical contradiction* that subsisted in the usual theory.

Superluminal velocities could be important for the initial, inflationary expansion of our Universe, but it is then necessary to include gravitational effects. This requires a modifiable metric of space and time. The quantum of length is then not a universal constant any more[5]. Some effects of quantum gravity can probably be treated at a larger scale, in terms of Planck's length. This leads to a foamy structure of space and time, where surfaces are quantized in terms of the square of the Planck length[4]. Similar ideas had been expressed already at a much earlier stage[7]. It should be noted that the walls of these bubbles would have a continuous structure, while we are considering a lattice for the possible results of space-time coordinates, and this lattice is independent of the gravitational constant G.

Although the generalized energy-momentum relation (6) applies to free particles, it allows us to predict what happens when a material particle is subjected to a force. We know, indeed, that the group velocity $v = v(p,m_o)$ can be deduced from (6), since $v = dE/dp$. We can thus calculate the acceleration dv/dt when the value of $F = dp/dt$ is known. We get then[5] *a generalization of Newton's law of motion*, $dv/dt = F/m$, where F defines the magnitude of the applied force, while m is the "inertial mass", which depends in a specific way on the velocity v, even when $v > c$. It is also possible to treat the case of motions along any direction with respect to the chosen spatial reference axes[5].

The objection that space-time quantization would be incompatible with the Lorentz transformation for space-time coordinates is not valid. It results from the antiquated concept

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[5] A. Meessen: Spacetime quantization, elementary particles and cosmology, Foundations of Physics **29**, 281-317 (2000) and http://www.meessen.net/AMeessen/STQ/STQ.pdf

[6] A. Einstein, B. Podolsky and N. Rosen: Can quantum-mechanical description of physical reality be considered complete? Phys. Rev. **47**, 777-780 (1935).

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of absolute space and time, suggesting that there should exist a unique space-time lattice or at least, that the result of an ideally precise measurement performed in one frame has to coincide with the result of an ideally precise measurement performed in any other frame. That's not necessary, according to general quantum mechanical principles. We insisted therefore already at the outset on the fact the space-time quantization is not concerned with an intrinsic structure of space and time, but with *possible results of measurement.* They depend on the chosen experimental set-up and in different reference frames, we are performing *independent measurements.* It is necessary, however, that the form of the energy momentum-relation (6) is preserved for all inertial frames. This calls simply for a generalized Lorentz transformation[8].

Since Heisenberg had considered the possible existence of an elementary length[9], we submitted in 1973 all our published papers and the projected paper about relativistic invariance[8] to his scrutiny. He had no objection, but advised to search for applications that can establish contact with experimental results. The privileged field for such an attempt could be *elementary particle physics,* since we know that there are various types of elementary particles, with properties, like *color, strangeness, charm or beauty,* and corresponding quantum numbers, but we don't understand their physical origin. Stationary energy levels are characterized in atomic, molecular and nuclear physics by specific spectroscopies, but they can always be explained in terms of quantum-mechanical laws. The "new spectroscopy" of elementary particle physics indicates the existence of a *deeper-lying level of reality.* It has to be governed by rational laws, but quantum mechanics alone doesn't give access to this *terra incognita.* Why do elementary particles have "internal degrees of freedom" that can't be explained by the usual theory? Does this result from the continuum assumption? Could a finite value of the smallest measurable distance (a) provide an explanation?

We tried to answer this question by following Dirac's procedure. When he replaced the Gordon-Klein equation (4) by *an equivalent set* of first order differential equations, he discovered already some internal degrees of freedom (spin-up and spin-down, as well as particle and antiparticle states for electrons and similar particles). When we did this for the corresponding finite difference equation, we found *new* internal degrees of freedom[10], but it took some time to discover their profound meaning and a simpler approach[5].

Generalized space-time quantization and elementary particles

Let's reconsider the measurement of the coordinate x in a given inertial reference frame. We can freely choose the direction, the orientation and the origin of the x-axis. Once this is done, an ideally precise measurement of the position of a point-particle will yield a particular value "x", but we can immediately assert that "-x" is also a possible result. The orientation of the x-axis has been chosen in a completely arbitrary way, indeed, so that spatial symmetry has to be insured. Since the distance 2x between x and -x has to be measurable, $2x = Na$, where N is an integer number. Thus $x = Na/2$, but N can be an even or an odd number. As indicated in figure 2, this yields two possible spectra: $x = 0, \pm a, \pm 2a, \pm 3a, \ldots$ or $x = \pm a/2, \pm 3a/2, \pm 5a/2\ldots$

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[8] A. Meessen: Spacetime quantization, generalized relativistic mechanics and Mach's principle: Foundations of Phys. **8**, 399-415 (1978)

[9] W. Heisenberg : Über die in der Theorie der Elementarteilchen auftretende universelle Länge, Ann. Phys. **32**, 20 (1938), Die Grenzen der Anwendbarkeit der bisherigen Quantentheorie, Z. Phys. **110**, 251 (1938). Quantum theory of fields and elementary particles, Rev. Mod. Phys. **29**, 269 (1957).

[10] A.Meessen : Quantification de l'espace-temps et généralisation de l'équation de Dirac, Ann. Soc. Sc. Bruxelles, **84**, 267-275 (1970), Quantification de l'espace-temps et généralisation de l'équation de Dirac: interprétation du degré de liberté additionnel, ibida, **85**, 204 (1971), A finite difference Schrödinger equation, ibida, **86**, 89-100 (1972), **88**, 71 (1974),

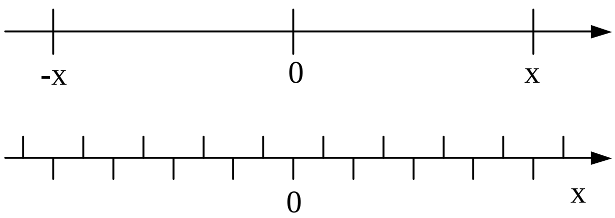

Figure 2: Ideally exact measurements of the coordinate x yield two different spectra of possible values.

This twofold quantization is valid for all (x,y,z,ct) reference axes. There are thus several space-time lattices of lattice constant *a*. The *"normal lattice"* contains the origin of the chosen reference frame, but there are *"inserted lattices"*, displaced by a/2 along one or more reference axes. The usual ψ function, defined on a space-time continuum, should now be replaced by a ψ function that is defined on all these lattices. Nevertheless, we can consider ψ functions that are respectively defined on the normal lattice and on inserted lattices. For every one of these lattices, the possible variations of ψ functions are described at the smallest possible scale by second order finite differences, like (5). Since they require only that we consider intervals that are equal to *a*, we get the same energy-momentum relation (6) for everyone of these space-time lattices.

We can say that E and p define "external degree of freedom", allowing us to distinguish *different states of motion* from one another for any type of particles, but we are looking for "internal degrees of freedom" that would allow us to distinguish *different types of particles* from one another. This is possible when we consider the ensemble of ψ functions that are defined on different space-time lattices of lattice constant *a*. Figure 3 represents what happens along the x-axis. The left part shows how the probability to find the particle at any particular point varies progressively along the x-axis. This probability has to vary in such a way that a smooth transition to the usual continuum approximation is possible.

Since the probability is equal to *the square* of the amplitude A of the ψ-vector, it follows that the orientation of the ψ-vectors could be inversed on the inserted lattice with respect to the normal lattice, as indicated in the right part of figure 3 for a particular point of the inserted lattice. The progressive variation of the phase angle φ of the ψ-vectors defines the state of motion. It is specified by the variables p and E, but this doesn't exclude that the angle φ can be increased or decreased everywhere on the inserted lattice with respect to the normal lattice by an integer multiple of 180°.

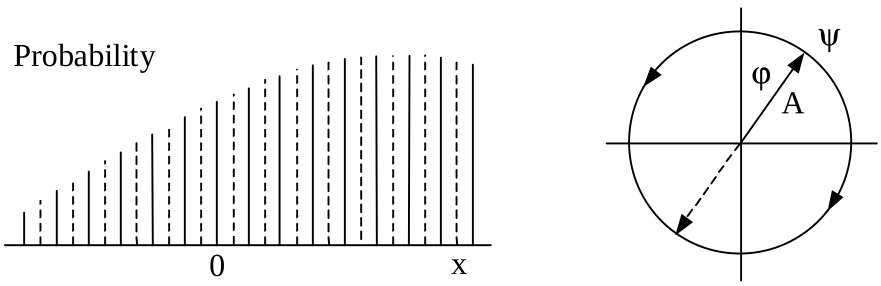

Figure 3: A smooth variation of the probability to find a particle at different points x allows for a discrete difference between the phase angles φ on the inserted lattice with respect to the normal lattice.

The angle φ = kx +… (where the other terms depend on y, z and ct), but kx will be increased *everywhere* on the inserted lattice with respect to the normal lattice by *u* times 180°, where u = 0, ±1, ±2, ±3,... This results from the fact that the generalized energy-momentum relation (6) contains sine functions[5], but the essential point is that the quantum number *u* defines a previously unknown *internal degree of freedom.* Positive or negative values of u correspond to rotations of the ψ-vector towards the left or the right. The value of *u* is completely independent of the value of k and thus of the momentum p along the x-axis.

This reasoning applies to all space-time coordinates (x,y,z,ct) in any particular, but arbitrarily chosen inertial reference frame. We get thus *a set of four new quantum numbers:* (u_x,u_y,u_z,u_{ct}). Independently from one another, they can have the values 0, +1 or –1, for instance. Different combinations of these values will then define different types of elementary particles or possible "particle states". Since a truly elementary particle cannot have any constitutive parts, it is a single point, but even point-particles can be distinguished from one another, since the associated ψ-vectors can vary in space and time according to *different patterns.* We couldn't be aware of these features as long as we believed that space-time is continuous. The actual size of *a* is irrelevant. It is only necessary that a is not zero.

Does this picture really account for known facts? "Ordinary matter" is the stuff we are made of and that constitutes all material things around us, up to extremely far galaxies. The corresponding elementary particles are represented in figure 4 by lattice points in a three-dimensional space, where the (u_x,u_y,u_z) quantum numbers can be equal to 0, ±1, while $u_{ct} = 0$. We can chose three orthogonal base-vectors, to get a lattice of cubic symmetry, and we can choose a particular order for the names we assign to these axes. Since this order is arbitrary, it is obvious that we should consider the average value

$$Q = (u_x+u_y+u_z)/3$$

Q defines the *electric charge,* expressed in terms of the quantum of charge e. This can also be justified by generalizing the theory of electromagnetic interactions[5].

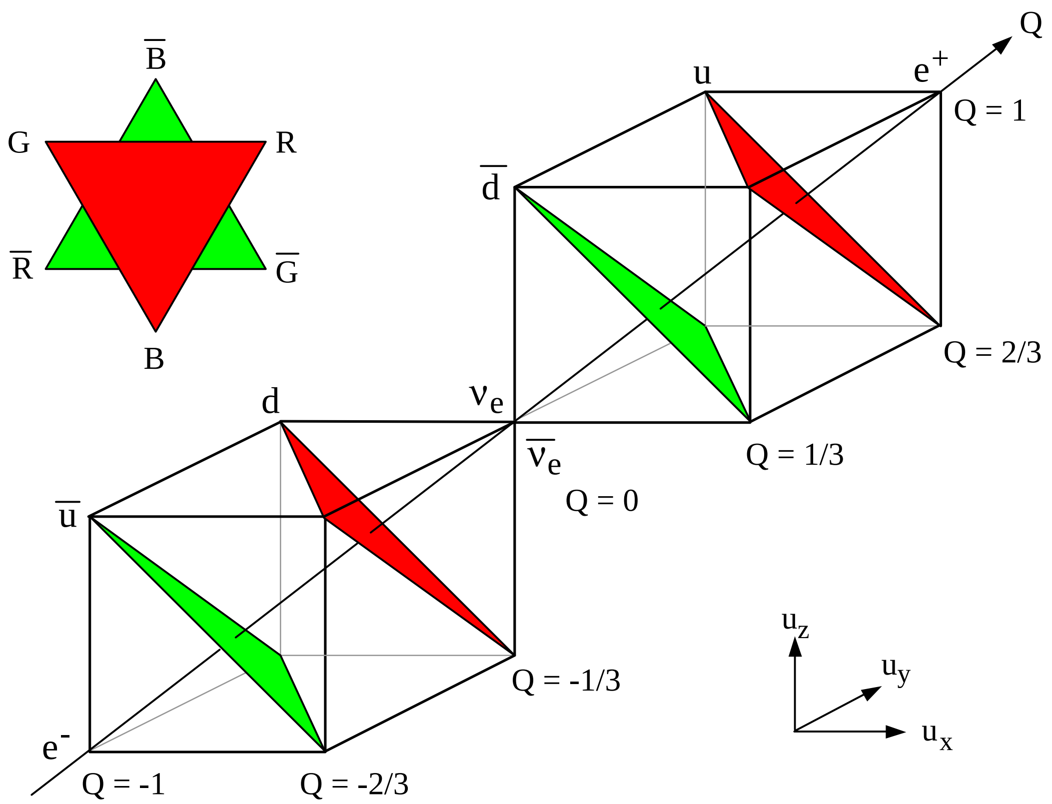

Figure 4: Elementary particles of ordinary matter are represented by points, defined by three quantum numbers (u_x,u_y,u_z) that can have the values 0, +1 or –1. Quarks and antiquarks correspond to the edges of the triangles, which are equilateral when seen along the Q axis.

The electron e^- corresponds to $(u_x,u_y,u_z) = (-1,-1,-1)$ and the positron e^+ to (1,1,1). The electron neutrino ν_e and its anti-particle are both characterized by (0,0,0), with Q = 0. There are two different types of quarks. The *up-quark* has three possible states: (0,1,1), (1,0,1) or (1,1,0), represented by points that are situated at the edges of the upper triangle. The resulting charge is Q = +2/3. The down-quark *d* has also three possible states: (-1,0,0), (0,-1,0) and (0,0,-1), with Q = -1/3. Although the naming of the (u_x,u_y,u_z) reference axes is arbitrary, we can distinguish three different states for up- and down quarks. By analogy with the three fundamental colors for human color vision, they are called *red, green and blue* (R, G, B) colors states. The sign of all u values is reversed for antiparticles. This changes the sign of Q, but also the color states. Anti-quarks have anti-colors.

This appears more clearly when we look at the triangles along the Q axis. As shown by the insert of figure 4, we get then two superposed equilateral triangles for the u and d quarks, and two centrally symmetric triangles for their antiparticles. It has been experimentally proven that *u and d quarks can only have three different color states.* This is now a consequence of the fact that space is three-dimensional, and provides a strong argument in favor of the proposed theory. We can also understand why nucleons and similar particles contain *three quarks with different colors.* The three spatial dimensions are physically equivalent.

So far, we considered only a group of particles, characterized by $u_{ct} = 0$. There are two other *generations*, with an identical internal structure, resulting from the (u_x,u_y,u_z) quantum numbers, but they are characterized by $u_{ct} = \pm 1$ instead of $u_{ct} = 0$. They are known to exist, but they had to be created by means of energetic particle collisions. In figure 4, we have then to replace the $e^\pm$ particles by a $\tau^\pm$ or $\mu^\pm$ particles, while the up and down quarks are respectively replaced by *charmed and strange* quarks or *top and bottom* quarks.

Fermions, bosons and new particles

All elementary particles of figure 4 and the analogous particles with $u_{ct} = \pm 1$, are *spin 1/2 particles.* The spin is very similar to a classically known property, the angular momentum vector, but in quantum mechanics, it describes the symmetry of the ψ function around a given spatial axis. This could be done already in the continuum approximation, and explains why the spin of a particle is independent of the values of the (u_x,u_y,u_z,u_{ct}) quantum numbers. Particles that have a half-integer spin (1/2, 3/2,…) are called *fermions*, while particles of integer spin (0, 1,…) are *bosons.* Particle states of fermions and bosons will respectively be designated by (u_x,u_y,u_z,u_{ct}) and $[u_x,u_y,u_z,u_{ct}]$. Since all known spin 1 particles correspond to $u_{ct} = 0$, they can simply be characterized by $[u_x,u_y,u_z]$ with the previous definition of Q.

Photons correspond to [0,0,0]. They can exist as real particles and are responsible for electromagnetic interactions, resulting from an exchange of virtual photons. Weak interactions are due to an exchange of $W^\pm$ and Z *weakons.* They are respectively characterized by [1,1,1], [-1,-1,-1] and [0,0,0]. Only points that are situated on the Q axis will represent these particles. In figure 4, the $e^\pm$ particles would thus be replaced by the $W^\pm$ bosons, but in the theory of electro-weak interactions, they are considered together with photons and Z particles.

Strong interactions are mediated by virtual *gluons.* They are electrically neutral but are characterized by [0,1,-1] and permutations of these quantum numbers. This allows for 6 colored quarks, characterized by *a color and an anti-color*. We can thus get, for instance, the following transformation: a red u-quark is annihilated, to create a green u-quark and a red-anti-green gluon ($R = G + R\overline{G}$). Since the u_x, u_y and u_z quantum numbers are separately conserved, this process can also be represented by (0,1,1) = (1,0,1) + [-1,1,0].

There are also $(R\overline{R})$, $(G\overline{G})$ and $(B\overline{B})$ states, corresponding to [0,0,0], but one considers only two independent mixtures of these states. The resulting 8 different types of gluons would be presented in a figure that is analogous to figure 4, by points that are situated in a plane that is perpendicular to the Q-axis, at Q = 0. The colored gluons would then be situated at the edges of a hexagon and the color-neutral gluons at its center.

Now it becomes obvious that we can expect the existence of *other types of elementary particles.* There should exist spin 1 bosons that are analogous to quarks and spin 1/2 fermions that are analogous to gluons, as well as other particles, with higher values of the u quantum numbers. This opens the road to new physics, beyond the "standard model", *without postulating the existence of additional dimensions,* as required by string theories. There are six particle states of type (0,1,-1), with possible permutations of these quantum numbers. They can't exist in isolation, but can form composite particles. Since they are electrically neutral and analogous to quarks, we propose to call them *"narks".* The resulting composite (spin ½) particles are called *"neutralons",* since they are analogous to nucleons.

Previously[5], we wanted to minimize the number of new particles, but in addition to the 6 colored narks, it is advantageous to consider also $R\overline{R}$, $G\overline{G}$ and $B\overline{B}$ narks, corresponding to [0,0,0]. Various types of narks can then be bound together by exchanging gluons, but statistically, all colors have to be present in equal proportions. This allows for the formation of neutralons, containing 1, 3, 5 or 7 narks. They are fermions like ordinary matter particles, but one can also get (spin 0 or spin 1) bosons.

Perhaps, we have already evidence concerning the existence of neutralons, since we know that our Universe contains huge amounts of *"dark matter".* This substance is composed of particles that are electrically neutral, but they have some mass, since they are subjected to gravitational effects. The actual mass of the individual particles is not yet known, but they have to be considered as *remnants of the initial big bang.* Neutralons are good candidates to account for dark matter. It is possible that their rest mass requires more energy for their creation than was available until now, but the Large Hadron Collider (LHC) could eventually produce this type of particles. Space-time quantization is thus experimentally verifiable.

Conclusions

It is *logically possible and physically useful* to generalize relativistic quantum mechanics, by considering the possible existence of a finite limit *a* for the smallest measurable distance. This quantum of length has to be *a universal constant* for all directions and all inertial reference frames, like c and h. The actual value of *a* remains unknown, but it is determined by *the total energy content of our Universe,* $E_u = hc/2a$. The usual continuum theories imply that the energy E of a particle could reach infinite values, but this is not necessarily true. On a purely logical level, there exists a strong indication that the quantum of length should be finite, since that would solve the EPR paradox. This is related to the fact that *superluminal velocities are not forbidden anymore when the smallest measurable distance a is not 0.* We also get then a better definition of inertial reference frames.

The generalized theory contains three universal constants: c, h and a. They define natural units for length, time and mass (or energy) measurements, so that these observables are now defined without any ambiguity for the whole Universe. The constants c, h and a are associated with three different types of restrictions that Nature imposes on our measurements. These restrictions have to be taken into account in physical laws, but we don't know why the constants c, h and a do have the particular values we observe in our Universe.

Because of the restrictions that Nature imposes on our measurements, it is now obvious that *physical laws are statements concerning the knowledge we can get about reality*. It is also important to note that these restrictions allow us to get more detailed knowledge. This appeared already through the fact that the theory of relativity disclosed the existence of a rest energy m_oc^2 for any material particle of given rest mass m_o. In quantum mechanics, we express the knowledge we can get about a particle by means of ψ-vectors. They provide *an extraordinarily flexible epistemological tool*. It allows us to calculate the probability to find the particle at any particular point in space and time, but also to get detailed probabilistic information about the possible states of motion (external degrees of freedom).

It is even possible, when *a* is not 0, to distinguish different types of particles from one another by means of different patters of variation of the ψ-vectors in space and time (internal degrees of freedom). These features are characterized by four *new quantum numbers,* (u_x,u_y,u_z,u_{ct}) that can have positive or negative integer values. The resulting scheme for the classification of elementary particle states accounts for all known particles and predicts the existence of other elementary particles. One type of these particles could constitute *dark matter,* known to exist in our Universe in much greater proportions than ordinary matter.

Although it may be difficult to abandon the familiar concept of a space-time continuum, the basic assumptions are in line with the general trend of the evolution of physics. Einstein insisted[11] on the "essentially constructive and speculative nature of thought and more especially of scientific thought". For physical theories, the "naturalness" or "logical simplicity" of the premises is very important, as well as the "inner perfection" of all these theories. We applied these rules to establish the foundations of space-time quantization, to test its logical consistency and to show that it has the advantage of removing a severe contradiction that subsisted between relativity and quantum mechanics.

The "external confirmation" is essential, of course, since physical theories should not only to be *correct*, but also *true.* This means that they have to be in conformity with observed facts. Their objective is, indeed, to describe what actually happens or could happen in Nature. In regard to the *creation* of a new theory, Einstein warned however against "the prejudice… that facts by themselves can and should yield scientific knowledge without *free conceptual construction".* The development of space-time quantization was mainly motivated by a search of greater harmony between various ideas and facts that constitute the basic tissue of physics.

It is very astonishing that the concept of a finite limit for the smallest measurable distance seems to provide the key for understanding the existence of different types of elementary particles. It is particularly amazing that *all fundamental building block of reality can be viewed as different kinds of excitations of space and time.* The Universe we are living in becomes more understandable, but at the same time we have to agree with Einstein, who said[12]: *"The most incomprehensible thing about the world is that it is comprehensible".* Why is there such a highly remarkable rationality? Where does it come from?

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