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Spacetime Quantization,
Elementary Particles and Cosmology

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Relativistic quantum mechanics is generalized to account for a universally constant *quantum of length a*. Its value depends on the total convertible energy content of our universe: $E_u = hc/2a$. The eigenvalues of all (x, y, z, ct) coordinates are integer or half-integer multiples of *a* in every particular inertial frame. There are thus several spacetime lattices of lattice-constant a: the "normal lattice" contains the origin of the chosen frame, while "inserted lattices" are displaced by a/2 along one or several reference axes. States of motion are defined by possible variations of ψ-functions on any one of these lattices. Particle states are defined by their relative phases, specified by *four new quantum numbers,* u_x, u_y, u_z, u_{ct} = 0, ±1, ±2,... They account for all known elementary particles and yield a natural extension of the standard model. Space-time quantization solves also the EPR paradox and other difficulties that subsisted in the usual continuum theories. It defines inertial frames and is related to cosmology.

1. Introduction

Newtonian mechanics implied three basic assumptions: (i) there exists an absolute space and time, (ii) particles move along well-defined trajectories and (iii) this happens in a space-time continuum. Since two of these postulates were abandoned during the first decades of this century, we wonder if the continuum assumption is a logical necessity or not. We have even to question its validity, for a fundamental reason.

The development of relativity and quantum mechanics disclosed, indeed, that *Nature can impose restrictions on our measurements.* They were related to the existence of two universal constants: *c* and *h*. Relativistic quantum mechanics takes into account both restrictions, but uses *differential* field equations. Since all physical laws have to be verifiable, at least in principle, these equations imply that it should be possible to measure always smaller and smaller intervals of space and time, without any finite limit.

The validity of this assumption has only been tested [1] up to distances of the order of 10^{-21} m. To be realistic, we should consider the value *"a"* of the smallest measurable distance as a yet unknown empirical parameter, instead of assuming *a priori* that $a = 0$.

We require only that the length a has to be a universal constant in all inertial frames, like c and h, but we are then confronted with two very fundamental problems. (i) Is it *possible* to construct a theory that contains c, h and a, without running into logical inconsistencies when $a \to 0$? (ii) Would such a theory be *useful*, by providing a better understanding of physical reality? Both questions will be answered in a positive way.

1.1. Short History

The continuum assumption entered *Euclidean geometry* through the postulate of "infinite divisibility of any line segment" that accompanied the famous postulate on parallel lines. Although both statements were based on physically uncontrollable extrapolations, they were viewed as evident truths, in the sense that alternative assumptions seemed to be unacceptable. It has even been claimed [2] that Euclid's postulate on parallel lines is provable, *ab absurdo,* but Lobachevsky, Bolyai and Riemann discovered, between 1829 and 1854, that it is possible to construct *logically consistent* non-Euclidean geometries. What would happen if we tried also to abandon the continuum assumption?

Clifford [3] considered already in 1870 a modification of Newton's laws of motion, but without changing the other postulates of classical mechanics. He simply endowed absolute space and time with a "discrete structure" and assumed that point-particles can only appear (and exist) at the resulting lattice-points. They would act like lights that can go on and off, one after another. This concept of "discontinuous motions" reappeared after the development of relativity [4], now combined with the idea of a highest possible velocity c, but Einstein modified physics in a much more profound way.

He recognized that (ideally precise) measurements of space and time intervals are subjected to *a universal restriction.* We can only get results that are related to one another in such a way that the velocity of light in vacuum has *the same value c* for any direction in all inertial reference frames. Heisenberg transposed this idea, by requiring that motions of atomic electrons should only be described in terms of *possible* results of measurement. Using spectroscopic data, he constructed matrix-mechanics, where the concept of space-time coordinates was generalized to account for the older "quantization rules". After the development of wave-mechanics, Heisenberg formulated the famous "uncertainty relations", demonstrating very clearly the existence of *an other universal restriction.* A particle can only be localized in space and time with a precision that depends on the accepted uncertainty about the momentum and energy of the particle, when these observables are defined in any given inertial frame by wave-properties and Planck's constant *h*.

The prominent event that shaped 20th. century physics was thus the very surprising, but nevertheless twofold discovery that *Nature can impose restrictions on our measurements*. This feature modified even the very status of physical laws. Instead of being direct statements about reality, they are assertions concerning the *knowledge* we can get about it. This knowledge results from measurements that are subjected to universal restrictions. They have thus to be included in the formulation of physical laws. Relativistic quantum mechanics combines the effects of *c* and *h*, but there are several signs, indicating that Nature could impose *a third restriction*.

Heisenberg's ideal gamma ray microscope allows us to determine the position of an electron with increasing precision by reducing the wavelength of scattered photons. Since the Compton effect increases this wavelength, the uncertainty about the position of *low-energy* electrons is subjected, however, to an absolute limit: $\Delta x \geq h/m_o c$. This restriction has to be added to Heisenberg's uncertainty relations [5]. It combines *c* and *h*. Landau and Peierls [6] proved that in general $\Delta x \geq hc/E$ for material particles of energy $E \geq m_o c^2$. This is less restrictive, but these authors were very concerned about the use of *classically* defined space-time coordinates in relativistic quantum mechanics.

Pauli [7] stated in a review of the basic principles of wave-mechanics that "only non-relativistic quantum mechanics is logically complete", since $\Delta x \geq 0$ when *c* is infinite. It should be noted, however, that even in relativistic quantum mechanics, it is possible to localize material particles with absolute precision when their energies can be infinite. Pauli did not stress the fact that this could be an approximation. Nevertheless, he vigorously expressed the conviction that new "limitations on the *possibilities of measurements* will be more directly expressed in a future theory, and this will be associated with essential and profound modifications of the basic concepts and formalism of the present quantum theory". Pauli stated even that "the concept of space and time at very small scales *needs* a fundamental modification".

This followed from the fact that the *calculated* values of some observables became infinite when continuum theories were extrapolated to arbitrarily small distances, although the *measured* values were finite, of course. This appeared already in classical electromagnetism. Quantum electrodynamics attenuated the divergence, but did not remove it. It was apparent, however, that the incoherence results from the assumption that *virtual particles can have infinite energies*. It was suggested [8] that the spectrum of possible wavelength should be cut off, but Pauli [7] objected that "a universally constant length can surely not exist, for reasons of relativistic invariance". It would be incompatible with the Lorentz transformation for spacetime coordinates, but we have to recall that this law was devised to insure the invariance of *differential* equations, containing *c*. The Lorentz transformation presupposes that $a = 0$. To allow for $a \to 0$, we have to modify the Lorentz transformation. How?

If there did exist a universally constant elementary length $a \to 0$, we would have to associate to every particular inertial reference frame a different spacetime lattice of lattice-constant *a*. Some authors [9] considered a *sub-group* of Lorentz transformations that would establish a correspondence between particular lattice-points in different inertial frames. This is not sufficient, since it would restrict the choice of relative motions, but we can start with a generalization of relativistic quantum mechanics, to account for *c*, *h* and *a* by means of *finite-difference* equations [10]. It is then only necessary to verify if all their consequences are relativistically invariant. This procedure leads to a *generalized* Lorentz transformation [11], containing *c*, *h* and *a*.

The divergence difficulties were circumvented by the *renormalization procedure*. Some theoretically infinite values are simply replaced by the measured, finite ones, to get extremely good predictions for other observables. Feynman [12] considered this *semi-empirical* method as "sweeping the infinities under the rug". They are still there. Although the divergence difficulties can often be eliminated by a clever grouping of terms, they are basically related to the fact that virtual particles can have *infinite energies* in continuum theories. Feynman stated that in the past, such a problem indicated always that "some deeply held idea had to be thrown away.... I believe that the theory that space is continuous is wrong".

Einstein was also convinced that the present foundations of physics have to be corrected, although he used another argument. The EPR paradox [13] revealed that quantum mechanics allows for instantaneous communications between distinct and widely separated elements of reality, but this is strictly forbidden by relativity. The effects of *c* and *h* contradict one another! Einstein concluded: *"the quantum-mechanical description of reality is incomplete"*. He meant [14] that this theory "may well become a part of a subsequent one, in the same way as geometrical optics is now incorporated in wave optics", but it is only an approximation of a more general theory.

There is a third, more concrete argument for expecting an enlargement of the present theoretical framework. It follows from the progressive accumulation of a huge amount of *unexplained experimental results* in elementary particle physics. We were able to create a great variety of particles that don't exist in the surrounding world. We could even distinguish them from one another by means of observables, like "quark colors" or "strangeness", for instance, but they are empirically defined. We don't understand their physical origin and real meaning. Moreover, we discovered that quarks have *three* possible color states and that there are *three* completely analogous families of quarks and leptons, but we can't explain these highly astonishing facts. There has to exist an *underlying rationality,* as for atomic spectra at the beginning of the 20th. Century. The key to this world was the quantum of action *h*. Perhaps, we have now to add a quantum of length *a*, to get access to another quantum world, involving different rules.

Heisenberg [15] and March [16] thought already that the properties of elementary particles are probably related to the existence of an "elementary length". Blokhintsev [17] concluded from a review of elementary particle physics: "it is clear that we need *new physical concepts* and, accordingly, a new language more suitable to the inner nature of elementary particles than the one we have... We perhaps need only two or three words... (but) these words must be no less revolutionary than those that led to the creation of the quantum theory and the theory of relativity". Physics should be simple, indeed, but it was often difficult to change our vision, because of deeply rooted habits of thought.

Several authors have considered a generalization of relativistic quantum mechanics by means of finite-difference equations [18], but they used various assumptions and advanced diverging interpretations. It is thus necessary to justify any choice and to explore its consequences in great depth. *Lattice gauge theories* introduced important innovations [19], but it was recognized [20] that "a lattice formulation rather severely mutilates Lorentz invariance at the outset". It was thus assumed [21] that the lattice constant a $\to$ 0 at the end of all calculations. Space-time quantization would then be a purely formal tool, but the generalized Lorentz transformation shows that it could be physically real. It's important to clarify this issue.

1.2. Aims and Plan

Basically, we want to find out if the continuum assumption is a logical necessity or not. For this purpose, it is sufficient to construct a generalized theory that contains *c*, *h* and *a*, to check its internal consistency when $a \to 0$. Since the generalized theory should be coherent for any values of the empirical parameters *c*, *h* and *a*, we don't have to know the actual value of *a* in advance. The particular case where $a = 0$ would correspond to the usual theory, but this could be an approximation, like $c = \infty$ and $h = 0$ in classical mechanics. It is essential to note that these values allow for a *qualitatively different image of nature*. The concept of a "space-time continuum" is perhaps as wrong as the concepts of "absolute space and time" and "well-defined trajectories".

The existence of a finite, universally constant quantum of length *a* would imply the existence of a finite, universally constant quantum of time *a/c*. We have thus to construct a theory of "spacetime quantization". Even when this is *logically possible,* we have still to verify if it is *physically useful*. Would it remove paradoxes that subsisted in the usual continuum theories? Could it account for known, but unexplained facts? Are there other possible tests?

In section 2, we introduce and justify *the basic principles of space-time quantization.* It is shown that the continuum assumption (a = 0) is only necessary in classical physics (when h = 0). The differential Gordon-Klein equation is replaced by a finite-difference equation to account for all possible states of motion. This yields three basic results: (i) the total convertible energy of our universe is finite, (ii) superluminal velocities are possible at extremely high energies, and (iii) all space-time coordinates can be integer or half-integer multiples of a in any particular inertial frame.

In section 3, we establish contact with *experimental observations* in elementary particle physics. Truly elementary particles can be distinguished from one another, when $a \neq 0$, by means of possible variations of their ψ-functions in space and time. The resulting "spacetime code" accounts for all known elementary particles, bosons as well as fermions, and it yields a natural extension of the standard model.

In section 4, we explore the relation between space-time quantization and *the whole universe.* This provides further tests of the consistency and usefulness of this theory. It preserves relativistic invariance as well as causality, in spite of superluminal velocities. They solve the EPR paradox and account for signal transmission by wave-tunneling at faster than light velocities. The quantum of length is also related to the definition of inertial frames and to cosmology.

2. Space-time quantization

2.1. Roots of the Continuum Assumption

Why are we so deeply convinced that space and time are continuous? This could mainly result from educational conditioning, but there are *two rational arguments.* They were even of fundamental importance for geometry and mechanics.

Initially, it was not obvious for ancient Greek thinkers that space is continuous. It was quite attractive, indeed, to assume that everything is composed of small *indivisible* parts. The idea of an "atom of length" was even suggested by the general rule that it is only licit to add quantities of the same type to one another. A line should result from a juxtaposition of *line elements*, and not of points, unless there are finite intervals. The idea of "geometrical atomism" was abandoned, however, although "physical atomism" survived. The reason was so important, that we recall it in algebraic form.

Let's assume that there exists an atom of length $a \neq 0$, and that it has always and everywhere the same extension. The sides and the diagonals of a square have then respectively a length s = ma and d = na, where m and n are integer numbers. The Pythagorean theorem implies that $n^2 = 2m^2$. Since the square of an odd number is always odd, n is an even number. Setting n = 2p, we get $m^2 = 2p^2$. Thus, m is also even, but when n and m are both divisible by 2, we cannot assert that *"a"* is the smallest possible length. It was so astonishing that $d/s = \sqrt{2} \neq n/m$, that the square root of 2 was said to be an *irrational* number, but the essential result was that the postulate of "infinite divisibility of any length" seemed to be the only possible one.

This is not true, since no distinction was made between measured and calculated observables. In principle, we can measure two sides of any triangle with absolute precision, and the resulting values could be quantized. The length of the third side is then implicitly known. It can be deduced by logical rules. The ancient justification of the continuum assumption is obsolete when *only measured lengths are quantized.*

There was another, more intuitive justification of the continuum assumption. Daily experience suggests, indeed, that the constituting parts of material objects have a *continual existence.* This required that "time is continuous." It is also natural to think that point-like particles have *always a well-defined position,* even at those instants where they are not observed. It follows then from their continual existence that they can only move from one point to another point, by passing through a continuous array of intermediate points. Their trajectories are continuous lines and "space is continuous".

Quantum mechanics is based on a different concept of motions, since we consider a *probability distribution* that changes in space and time. This idea is compatible with the continuum assumption, but does not require it. At any given instant t, we could define the probability distribution $|\psi|^2$ on *a spatial lattice* of lattice-constant *a*. This means that a point-particle (or the center of mass of an extended material body) can only be localized with absolute precision at anyone of the allowed lattice-points. The quantum-mechanical energy-momentum variables would then be completely undetermined, but this restriction is only added to the new one, concerning ideally precise distance measurements. The average position at an instant t will be calculated by means of $|\psi|^2$ and can thus fall between lattice-points. It will even vary in a continuous way when the time variable is continuous.

This happens in the approximation where $c = \infty$, since the quantum of time $a/c = 0$, but the rest-energy of material particles is then infinite. Creation and annihilation processes are impossible. The continuity of time is thus still related to the continual existence of material particles when it is considered as an approximation. Moreover, it appears that the usual concept of "continuous motions" is compatible with the possible existence of a quantum of length $a \neq 0$, but only when $h \neq 0$, since it requires a probabilistic description of motions. The mental image of a "spacetime continuum" belongs to classical mechanics, but it survived the quantum-mechanical revolution, since it was deeply ingrained in our brains through an unconscious analysis of daily experience. Let's try to think in a different way.

2.2. The Basic Postulate

The smallest measurable distance a is a universal constant for any direction in all inertial frames, like c and h.

This includes the case where $a = 0$, but when $a \neq 0$, we can view all ideally precise distance measurements as resulting from *successive juxtapositions* of the smallest measurable distance *a*. When they are performed along a given x-axis by starting at its origin x = 0, we can only get x = na, where n = 0, ±1, ±2,... This applies also to y and z, measured along two other directions in three-dimensional space. The eigenvalues of the (x, y, z) coordinates will thus define a lattice of lattice-constant *a* in any particular inertial frame. But *space remains homogeneous and isotropic,* since the origin and the orientations of the reference axes can be chosen in a completely arbitrary way.

The time variable t is measurable by determining the distance ct that a light pulse travels during the time t along any fixed, but *arbitrarily chosen direction* in a given inertial frame. The length ct will be an integer multiple of *a*, when the measurement starts at the instant t = 0. The eigenvalues of all four (x, y, z, ct) spacetime coordinates define a "spacetime lattice" of lattice-constant *a*. Spacetime lattices can be displaced and move relative to one another, but the lattice-constant is always the same.

Snyder [22] proposed an other quantization scheme, based on a formal transposition of the usual theory for orbital angular momentum components (L_x,L_y,L_z). The (x, y, z, ct) coordinates were assumed to depend on four continuous variables and their conjugate momenta, with analogous commutation rules. *Only one* of the (x, y, z, ct) coordinates could then be measured with absolute precision and quantized in terms of *a*. Instead of introducing occult variables, we simply maintain the usual idea that the (x, y, z, ct) coordinates are basic observables, allowing for *independent* measurements.

2.3. Modified Laws of Motion

Let us consider a point-particle (or the center of mass of a material system) that is freely moving in a given inertial frame. Its possible positions x can be measured along the direction of motion, defining the x-axis. The possible (quantum-mechanically unmixed) states of motion are then specified by perfectly harmonic functions:

$$\psi = A\, e^{i(kx-\omega t)}, \qquad \text{with} \qquad E = \hbar\omega \quad \text{and} \quad p = \hbar k \tag{1}$$

E and p are new (non classical) energy and momentum variables, but it was assumed that these observables are still related to one another by *Einstein's relation*

$$(E/c)^2 - p^2 = (m_o c)^2 \tag{2}$$

This is compatible with (1), when ψ satisfies the *Gordon-Klein equation*

$$\partial_x^2 \psi - \partial_{ct}^2 \psi = (m_o c/\hbar)^2 \psi \qquad (3)$$

The second-order partial derivatives of ψ with respect to x and ct are always and everywhere related to one another in the same way, depending on the rest-mass m_o of the particle. To allow for $a \neq 0$, we adopt the *rule of correspondence*

$$\partial_x^2 f(x) \;\rightarrow\; D_x^2 f(x) = \frac{f(x+a) + f(x-a) - 2f(x)}{a^2}$$

f is any physically acceptable function of x and other variables. Since a weighted sum of similar expressions with intervals $\Delta x = na$, where $n = \pm 1, \pm 2, \pm 3, \ldots$ would yield the same limit when $a \rightarrow 0$, we have to justify this rule. Simplicity is not enough. To generalize (3), we have to consider the *physical meaning* of this law. It is equivalent to Einstein's relation (2) for "sharply-defined values" of E and p. The general solution of (3) is a linear superposition of particular solutions (1). The "average values" of E and p are then still related by (2). The wave equation (3) is even equivalent to Einstein's relation (2) for "local values" of E and p, defined by $E\psi = i\hbar\partial_{ct}\psi$ and $p\psi = -i\hbar\partial_x\psi$. This confers an intuitive meaning to quantum-mechanical operators and shows that (3) should be generalized at *the smallest possible scale.*

The energy E and momentum p are still defined by (1) when *a* • 0, since this yields simple *addition laws* for composite particles and simple *conservation laws* for creation and annihilation processes. This terminology is also used for phonons in crystal lattices. The local value of $p = \hbar k$ is defined by $D_x^2\psi = -(2/a)^2\sin^2(ka/2)\psi$, since this yields a sharply-defined value when $\psi = e^{ikx}$. In a similar way, $D_{ct}^2\psi$ yields the local value of $E = \hbar\omega$. When $a \neq 0$, we have thus to replace (3) by a finite-difference equation that is equivalent to the *generalized energy-momentum relation*

$$\sin^2(\pi a E/ch) - \sin^2(\pi a p/h) = \sin^2(\pi a E_o/ch) \qquad (4)$$

This is a new physical law, containing *c*, *h* and *a*. It applies to free motions of any particle or material system along the chosen reference axis. The second member is a constant, written in such a way that $E = E_o$ when $p = 0$. Since (4) should reduce to Einstein's relation (2) when $a = 0$ or $|E/c|$ and $|p| \ll h/2a$, the rest-energy $E_o = m_o c^2$, as in special relativity. The essential feature of (4) results from the fact that we can add integer multiples of π to the arguments of the sine-functions without changing the square of these functions. *The energy-momentum space has a periodic structure!*

The "first Brillouin zone", where |E/c| and |p| ≤ h/2a, is sufficient to specify all *possible states of motion*. Larger values of E/c and p (in the extended zone scheme) would define variations of ψ at a smaller scale than *a*, but they are irrelevant for the finite-difference equation, since it involves only values of ψ at lattice-points that are separated by *a*. Figure 1 shows some functions E = E(p) for positive values of E/c and p in the first Brillouin zone. The complete curves are symmetrical with respect to the E/c and p axes. For photons (m_o = 0), we get the usual law E/c = p, represented by the principal diagonal in figure 1. For material particles (m_o • 0), the curves coincide with Einstein's hyperbolas for E/c and p « h/2a, but they are distorted for increasing values of p and they shrink when $E_o \rightarrow E_u$ = hc/2a.

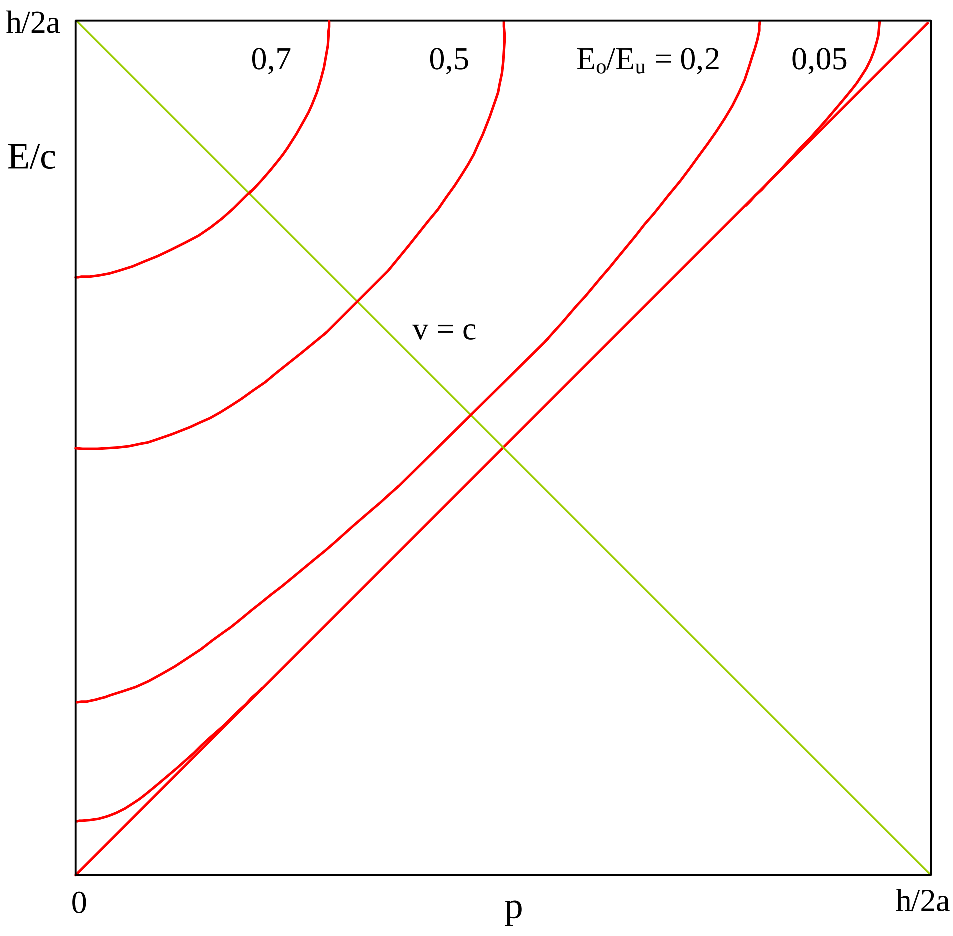

Fig. 1. The generalized energy-momentum relation for positive values of E and p. The rest-energy $E_o = m_o c^2$ *and the total energy content of our universe* E_u *= hc/2a.*

In the limiting case where $E_o = E_u$, there remains only one possible state of motion: p = 0. A material system that would have this rest-energy could only be in a state of rest. We could even say that (4) differs from (2), to allow for this particular feature, since it has an important physical meaning [10]: *the energy* E_u *is the total convertible energy content of our universe.* It is thus the highest possible energy of any particular system, and if a single material system would have a rest energy $E_o = E_u$, it could only be at rest (p = 0) in the chosen inertial frame, since there would be no energy left that could appear in the form of kinetic energy.

Einstein's relation (2) was based on the assumption that particles can have infinite energies in a relativistic world ($c \neq \infty$). To find out how (2) has to be modified when E_u is finite, we had to use ψ-functions ($h \neq 0$) and a space-time lattice ($a \neq 0$). We can directly justify that $E_u = hc/2a$, by looking for an operational definition of the smallest measurable distance [11]. We get $a = \lambda/2$, where λ is the smallest possible wavelength. It depends on the highest possible momentum $p = h/\lambda$, achievable with a given energy E, by using particles of zero rest-mass. By setting E = cp, we insure that the velocity of free photons is always equal to *c*, but $E \leq E_u$. Thus, $a = hc/2E_u$.

The classical (or quantum-mechanical average) velocity of a particle is the *group velocity* of the associated wave-packet. It is the velocity of the point where optimal interference is achieved, even when $a \neq 0$. Thus, $v = d\omega/dk = dE/dp = v(p)$, where p is the average momentum of the particle. It is sufficient to consider the slope of the E(p) curves in figure 1, to see that $a \neq 0$ allows for *superluminal velocities* when $E \geq E_u/2$. The "light barrier" is defined by the second diagonal. In the usual theory, it was rejected to infinity, but it is only a horizon. We enlarge thus our vision of reality.

Superluminal velocities are irrelevant for telecommunications and space travel, since $v = c$ for free photons and $E \ll E_u/2$ for material objects, but the possible existence of superluminal velocities is very important in regard to principles. It should be noted that superluminal velocities were not excluded by some basic principle of special relativity. It followed from the constancy of c and the (implicit) assumption that $a = 0$.

For free motions along an arbitrary direction with respect to given (x, y, z) reference axes, we have to use the *generalized Gordon-Klein equation*

$$\Sigma_j D_j^2 \psi - D_{ct}^2 \psi = \mu^2 \psi \qquad (5)$$

where j = x, y, z and $\mu = (2/a)\sin(\pi a E_o/hc)$. The *relativistic invariance* of this equation is automatically achieved, since the possible values of ψ are only related to one another for intervals that are equal to the universally constant quantum of length *a*.

2.4. Generalized Coordinates

Once we have chosen the direction and the origin of the x-axis, we are still free to orient this axis in two different ways, but physics cannot be affected by this choice. When "x" is a possible eigenvalue, "-x" is also one. Since the length 2x of the resulting interval can be measured by starting at one of its extremities, it has to be an integer multiple of *a*. Thus,

$$x = na, \qquad \text{where} \quad n = 0, \pm 1, \pm 2, ... \quad \text{or} \quad n = \pm 1/2, \pm 3/2, ... \tag{6}$$

The eigenvalues of x are integer or half-integer multiples of *a*. We get a *generalized coordinate*, as there exists a generalized angular momentum component $J_z = m\hbar$ for any given z-axis. This concept accounts for the spin, initially associated with rotating particles, but actually specified by the variations of ψ around a given z-axis. J_z is sharply-defined for a harmonic function $\psi(\phi) = e^{im\phi}$, where ϕ is the azimuthal angle around the z-axis. Since the probability distribution has to be single-valued,

$$|\psi(\phi+2\pi)|^2 = |\psi(\phi)|^2 \qquad \text{or} \qquad \psi(\phi+2\pi) = \pm\, \psi(\phi)$$

This yields a *classification* of ψ-functions by means of an operator that produces a complete rotation around the chosen z-axis with real eigenvalues ±1. The possible eigenvalues of $J_z = m\hbar$ are determined by the condition $e^{im2\pi} = \pm 1$. This allows for integer and half-integer values of m. The quantization of the coordinate x can be justified in a similar way. Consider the wave-packet $\psi(x') = \int c(k)\, e^{ikx'} dk$, where x' is a possible position of a point-particle. When $c(k) = e^{-ikx}$ and $-\pi/a \leq k \leq \pi/a$, while the distances x'-x are integer multiples of a, we get $\psi = 0$ for x' • x, but not for x' = x. It follows that c(k) is the eigenfunction of x in k-representation, but the periodic structure of the momentum space requires that the probability distribution $|c(k)|^2$ is periodic:

$$|c(-\pi/a)|^2 = |c(\pi/a)|^2 \qquad \text{or} \qquad \exp(i2\pi x/a) = \pm 1$$

The possible eigenvalues of x are given by (6). The same argument applies to all (x, y, z, ct) coordinates. When their eigenvalues are *integer* multiples of a, we get the "normal lattice", containing the origin of the chosen reference frame. When one or more of the (x,y,z,ct) coordinates are half-integer multiples of *a*, we get "inserted lattices", displaced by *a/2* along the chosen reference axes. This has important consequences.

3. Elementary particles

3.1. The Spacetime Code

We are accustomed to distinguish macroscopic objects from one another by means of their shape or internal constitution, but *true* elementary particles can't have any parts. They are single points. How is it possible to distinguish points from one another? They have no distinctive features, but everything that we can know about a given particle or type of particles is inscribed in its ψ-function. We have to *crack the code.*

We attribute always a vector in a complex space, $\psi = Ae^{i\phi}$, to *every* point of space and time where a point-particle could be localized in a given frame. When space and time are quantized, we have thus to define ψ on *all* space-time lattices of lattice-constant *a.* States of motion of a free particle are defined by the finite-difference equation (5) for any space-time lattice of lattice-constant *a.* A given state of motion is characterized by identical values of the energy-momentum variables for all these lattices. Although the probability distribution $|\psi|^2$ reduces also to a single, smoothly varying one in the continuum approximation, ψ-functions can oscillate with *equal or opposite phases* on inserted lattices with respect to the normal lattice. It should be noted that the experimental set-up physically privileges this lattice. It is thus possible to define different "particle states" by means of the relative phases of ψ-functions on different space-time lattices, associated with the chosen inertial frame.

Observable elementary particles are not truly elementary (point) particles, since they contain a core-particle, surrounded by a cloud of constantly emitted and reabsorbed virtual particles. The possible types of virtual particles are determined by the nature of the core particle. A classification of point-particles applies thus also to observable elementary particles. We will use the same symbols. The mass spectrum of observed elementary particles depends on the average effect of virtual particles and calls thus for heavy computer work with lattice gauge theories. The underlying concepts are simple and general, however, as in atomic and nuclear physics.

3.2. New Quantum Numbers

Let us consider a point-particle that is freely moving along the x-axis, with a sharply-defined momentum $p = \hbar k$. This means that $\psi = Ae^{ikx}$ on the normal lattice, while $\psi = U_x Ae^{ikx}$ on the inserted lattice, with $U_x = \pm 1$. When U_x characterises the particle state, independently of k, we can factorize U_x for any wave-packet. Instead of considering oscillations with *equal or opposite phases*, we will treat U_x as a vector of magnitude 1 that can rotate in a complex plane, but that has to be aligned with the real axis to get a well-defined particle state. Thus,

$$U_x = \exp(iu_x\pi) = \pm 1, \qquad \text{and} \qquad u_x = 0, \pm 1, \pm 2, ... \tag{7}$$

The quantum number u_x defines an "internal" degree of freedom, while k defines an "external" degree of freedom, corresponding to different states of motion along the chosen x-axis. It is remarkable and useful to note that u_x can be introduced by considering a general function $\psi = Ae^{ik'x}$, where k' belongs to the *extended* zone scheme, while x belongs to the normal or the inserted lattice. Thus, $k' = k+u_x(2\pi/a)$, where k is situated in the first Brillouin zone $(-\pi/a \le k \le \pi/a)$, and $u_x = 0, \pm 1, \pm 2, ...$ tells us if k' is situated in the first Brillouin zone or in a neighboring one, at the left or the right of the central zone. On the normal lattice, we get $\psi = Ae^{ikx}$, since x = na, where n is an integer number. On the inserted lattice, x = (n+1/2)a and $k'x = kx + u_x\pi$. This yields $\psi = U_x e^{ikx}$ with the definition (7).

Similar U-factors and u-quantum numbers can be introduced for all (x, y, z, ct) spacetime coordinates. Particle states are now defined by sets of (u_x, u_y, u_z, u_{ct}) quantum numbers with integer eigenvalues. Every set corresponds to a particular pattern for the possible oscillations of ψ-functions on the interwoven space-time lattices. For a given type of particles, this pattern is identical in the whole universe and for all inertial frames. Since it corresponds to a particular energy distribution, we can say that *elementary particles are excitations of space and time!*

The ψ-function of composite particles is a linear superposition of products of all ψ-functions for the constituent particles. Since their U-factors are factorized, we get *separate addition laws* for every u-quantum number. Interactions are governed by selection rules, resulting from non-vanishing values of matrix elements. They involve products of ψ-functions for initial and final states. Annihilation and creation processes are thus subjected to *separate conservation laws* for u_x, u_y, u_z and u_{ct}.

Since the spatial variations of ψ around a given z-axis, defining the spin, are independent of the relative phases of ψ on different space-time lattices, we get a *supersymmetry*. Every fermion state (u_x, u_y, u_z, u_{ct}) corresponds to a boson state, with the same set of u-quantum numbers. We represent this boson state by [u_x, u_y, u_z, u_{ct}].

Since the reversal of the orientation of the x-axis calls for a change of sign of the momentum variable k' in the extended zone scheme, it implies a change of sign of u_x. The parity operator P (changing the sign of x, y and z) reverses the sign of u_x, u_y and u_z, while the time inversion operator T changes the sign of u_{ct}. Charge conjugation C (transforming particles into their antiparticles) reverses the sign of all u-quantum numbers, in conformity with CPT invariance.

3.3. Quarks and Leptons of the Standard Model

When $u_{ct} = 0$, it is sufficient to consider the triplet (u_x, u_y, u_z) to define different fermion states. They are represented by points in a three-dimensional cubic lattice, but in figure 2, we consider only those lattice-points that characterize *quarks and leptons of ordinary matter*. These spin 1/2 particles constitute the first family of the standard model. The identification of particle states is based on the requirement that all (u_x, u_y, u_z) quantum numbers have opposite signs for particles and antiparticles, while the electric charge (in units e) is defined by

$$Q = (u_x + u_y + u_z)/3 \qquad (8)$$

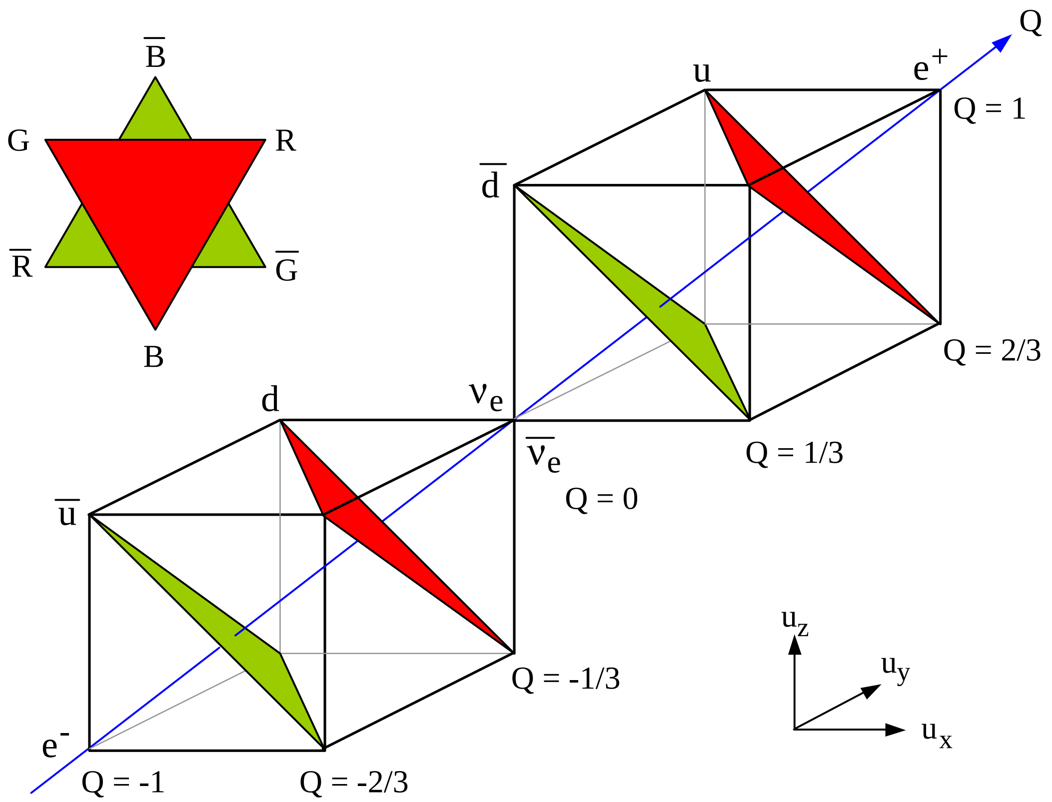

Fig. 2. The lattice points define elementary particle states for quarks and leptons of the first family in the standard model. They are characterized by u_x, u_y *and* $u_z = \pm 1$, *while* $u_{ct} = 0$. *The electric charge* $Q = (u_x+u_y+u_z)/3$. *The insert defines color states of quarks and antiquarks by means of a projection along the Q-axis.*

Electron and positron states correspond respectively to (-1, -1, -1) and (1, 1, 1). The electron-neutrino and its antiparticle are both characterized by (0, 0, 0). The *up-quark* has three possible states: (0, 1, 1), (1, 0, 1) and (1, 1, 0), with Q = 2/3. The *down-quark* has also three possible states: (-1, 0, 0), (0, -1, 0) and (0, 0, -1), with Q = -1/3. When the cubes of figure 1 are viewed along the Q axis, we get superposed equilateral triangles. For quarks, we can define three (R, G and B) color states by the directions of the edges of the corresponding triangle with respect to its center. Antiquarks have anticolors. The essential point is that *quarks have 3 color states, since space is three-dimensional.*

When a single quark is in a particular color state, it has an intrinsic memory of a spatial direction. This is not absurd, since the helicity of neutrinos (orientation of their spin with respect to the direction of motion) implies also an intrinsic spatial memory. Baryons are said to be "colorless", since they contain three quarks in different color states with equal weights. There is no preferred spatial direction. Leptons are automatically colorless, since u_x, u_y and u_z have equal values.

When we consider $u_{ct} = 0, \pm 1$ with the same possibilities for (u_x, u_y, u_z), we get *three families* of quarks and leptons. They are those of the standard model and are represented in figure 3 by means of u_{ct} and Q. The orientation of the u_{ct}-axis is chosen in such a way that $u_{ct} = +1$ for the top quark. This convention is analogous to the orientation of the Q-axis, to get Q = -1 for electrons. All (u_x, u_y, u_z, u_{ct}) quantum numbers have opposite signs for particles and antiparticles of the same type.

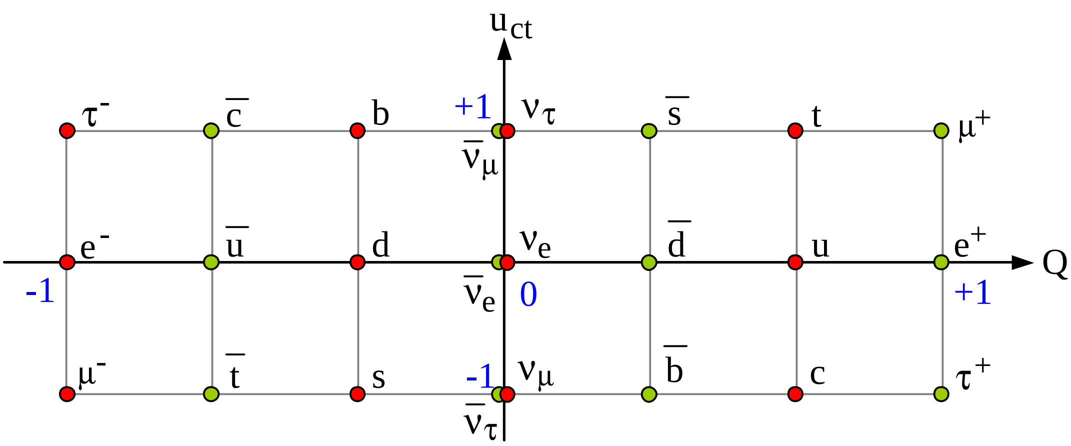

Fig. 3. The known families of quarks and leptons are characterized by Q and $u_{ct} = 0, \pm 1$. All u-quantum numbers of particles and antiparticles have opposite signs.

The electric charge is always defined by (8), without u_{ct}, since Q is revealed by interactions with *vector* fields. A formal justification of (8) will be given later on. The baryon number B is always defined by $Q = (B/2) + I_3$, when $I_3 = \pm 1/2$ for the quark or antiquak pair of the same family. This yields $B = \pm 1/3$.

3.4. Vector Bosons of the Standard Model

Quarks and leptons are spin 1/2 particles that can be converted into one another through the emission or absorption of spin 1 bosons, eventually accompanied by spin 0 bosons (scalar Higgs fields). Particle conversions within the first family ($\Delta u_{ct} = 0$) correspond to transitions from one lattice-point to another in figure 2. Color changes of quarks or antiquarks correspond to shifts along the sides of the equilateral triangles. A red u-quark, for instance, can be annihilated by creating a green u-quark and a *red-antigreen gluon*. This process is represented by (0, 1, 1) = (1, 0, 1) + [-1, 1, 0]. The red-antigreen gluon can be absorbed by a green d-quark, becoming red: (0, -1, 0) + [-1, 1, 0] = (-1, 0, 0). There are 6 bosons of type [-1, 1, 0], when we allow for permutations of these quantum numbers. These "color-changing gluons" account for strong interactions, represented by shifts that are *perpendicular* to the Q-axis.

Electroweak interactions are represented by shifts that are *parallel* to the Q-axis. Their carriers are γ, Z, W^+ and W^- vector bosons. A green u-quark can become a green d-quark through the emission of a W^+ boson: (1, 0, 1) = (0, -1, 0) + [1, 1, 1]. This W^+ can be absorbed by an electron, to create an electron-neutrino: (-1, -1, -1) + [1, 1, 1] = (0, 0, 0). $W^\pm$ bosons carry a charge Q = ±1, while γ and Z bosons are neutral and characterized by [0, 0, 0]. The photon is massless, but Z and $W^\pm$ particles acquire a mass through associations with spin 0 bosons.

We have also to expect transitions that are *oblique* with respect to the Q axis in figure 2. The carriers are then X and Y gluons of type [1, 0, 0] and [1, 1, 0]. They convert quarks into leptons and vice versa, so that (1, 1, 1) $\rightarrow$ (0, 1, 1) + [1, 0, 0] or e^+ $\rightarrow$ u + X, and (0, 0, 1) + [1, 0, 0] $\rightarrow$ (1, 0, 1) or anti-d + X $\rightarrow$ u. Antimatter becomes matter, while the baryon number B and the lepton number L are not conserved. In a representation of bosons states like that of figure 2, the X and Y gluons are analogous to quarks and antiquarks, while γ, Z and $W^\pm$ bosons correspond to leptons.

3.5. Beyond the Standard Model

Since u_{ct} = 0, ±1, we can also consider u_x, u_y, u_z = 0, ±1. This yields figure 4, with 12 new particle states for every family of quarks and leptons. The edges of the hexagon define 6 fermions of type (-1,1,0). Their existence is very probable, since bosons of type [-1,1,0] are known to exist. Both types of particles are characterized by a color and a different anticolor (figure 5a). Since fermions of type (-1, 1, 0) are subjected to strong interactions like *quarks,* but electrically *neutral,* let us call them *"narks".*

A projection of figure 4 along the Q axis yields also two large triangles, framing small triangles (figure 5b). Particles of type (1, -1, 1) with Q = 1/3, could be called "heavy d-quarks", to preserve the rule that particles have colors, while antiparticles have anticolors. We get two other large triangles, by extending figure 4 along the Q axis, to allow for u-quantum numbers equal to 0, ±1, ±2,.... This yields "heavy u-quarks" of type (0, 0, -2) with Q = -2/3 and their antiparticles.

Bosons of type [0, 0, -2] allow for particle-antiparticle transformations, like (0, 0, -1) = (0, 0, 1) + [0, 0, -2] for d-quarks. A similar transformation for u-quarks would require another boson: (0, 1, 1) + [0, -2, -2] = (0, -1, -1). The customary color representation yields 6 color-changing gluons and 2 independent combinations of red-antired, green-antigreen and blue-antiblue color states. The u-representation yields another classification of spin 1 bosons (see table I).

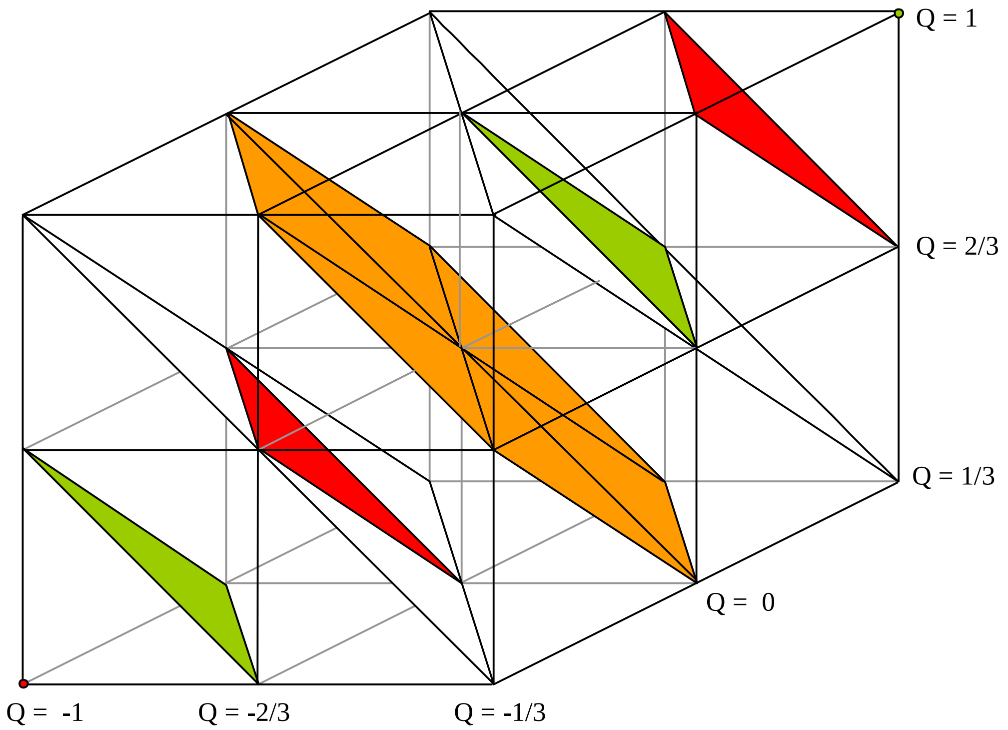

Fig. 4. An extension of the standard model, applying to fermions and bosons.

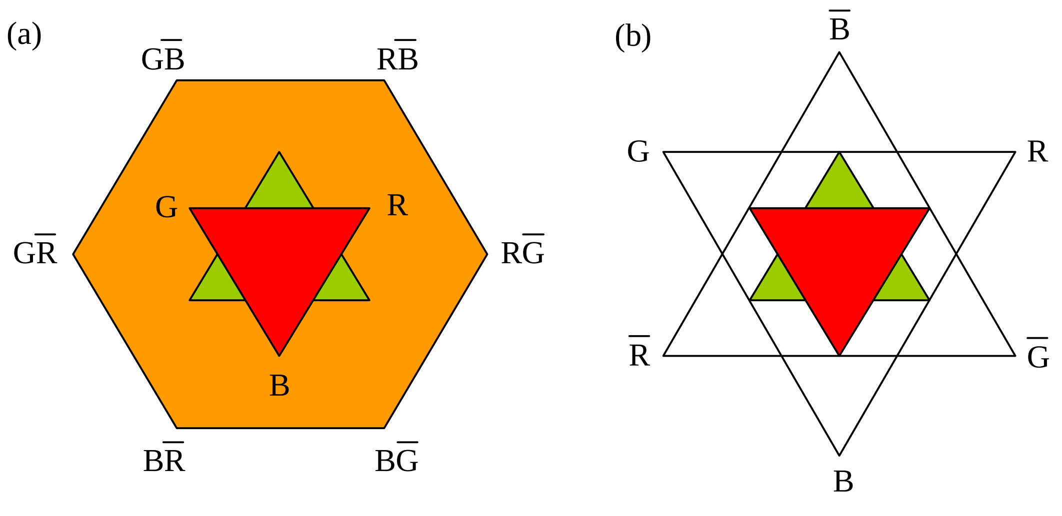

Fig. 5. (a) The color states of quarks, gluons and narks.
(b) The color states of heavy quarks and other gluons.

Table I: Classification of spin 1 bosons with $u_{ct} = 0$

Q= 0:	type [0, 0, 0] bosons (γ, Z) and type [-1,1,0] color-changing gluons,
Q= ±1/3:	type [1, 0, 0] and [1, -1, 1] X gluons and their antiparticles,
Q = ±2/3:	type [1, 1, 0] and [0, 0, 2] Y gluons and their antiparticles,
Q = ±1:	type [1, 1, 1] bosons ($W^{\pm}$),
Q= ±4/3 :	type [0, 2, 2] gluons,...

The famous LEP experiment concerning the decay of Z bosons [23] analyzed the reaction (e^-, e^+) $\rightarrow$ Z $\rightarrow$ (X, anti-X), where X could be a quark, a charged lepton or an undetected neutral particle. In the explored energy range, the measurements allowed only for three types of neutrinos (ν_e, ν_μ or ν_τ). This confirmed the concept of *three families* for the standard model, and therefore of $u_{ct} = 0, \pm 1$. It is not excluded that a nark-antinark pair could appear beyond some (yet unknown) threshold, but this pair would be rapidly converted into a neutrino-antineutrino pair [or a color-neutral nark pair] through transitions from opposite edges of the hexagon towards its center in figure 4. The Feynman diagram would contain a line for a color changing gluon, but the end product would be color neutral, and the width of the resonance could be modified.

Flavor changing transitions are possible with $\Delta u_{ct} = 0$, by means of flavor mixing intermediate states (box or penguin diagrams), but we have also to expect the existence of *flavored bosons,* with $u_{ct} = \pm 1$. They would allow for direct transitions between different families of the standard model, like u $\rightarrow$ s or $\mu^- \rightarrow \nu_e$ for instance.

The distinction between a "normal" spacetime lattice and "inserted" spacetime lattices is equivalent to the introduction of additional dimensions, but we don't have to leave ordinary space and time. *String theories* involve also additional (curled up) dimensions, so that particle states could be associated with different modes of oscillation of closed or open one-dimensional entities. This intuition can be carried over, by considering "twisted ribbons" instead of lines. First, we imagine two parallel poles, separated by the quantum of length *a* along the x-axis. Then, we imagine a ribbon that is attached to these poles, after twisting it in-between by u_x half-turns towards the right when u_x is positive (and towards the left when u_x is negative). We apply the same procedure to the y, z and ct axes. We can return to the first pole with the same number of half-turns for the same directions. We can even increase the size of this closed ribbon, by requiring only that the same number of twists applies to every step *"a"* along every spacetime axis.

This is a particular implementation of the spacetime code. It stores information in twisted ribbons to define fermion states (u_x, u_y, u_z, u_{ct}) or boson states [u_x, u_y, u_z, u_{ct}]. This accounts also for interactions, when Y portions of Feynman diagrams are viewed as separations or fusions of twisted ribbons. They are not simply attached to one another, but superposed, with conservation of the total number of half-turns for every elementary step along the chosen reference axes. The ribbons represent particle states, and should not be considered as physical entities.

3.6. Dark Matter

Several astrophysical and cosmological observations seem to indicate that more than 90% of the mass of our universe corresponds to *dark matter*. The constituent particles are unidentified, but they must have a finite mass, be electrically neutral and [practically be] subjected only to gravitational interactions with baryonic matter. They should be remnants of the Big Bang, since they are widely distributed in our universe and responsible for its sponge-like structure. To show this, it is sufficient to assume that dark matter behaves like an isothermal ideal gas. Its pressure $P = \rho kT/m$, where ρ is the mass density and m the (average) mass of the constituent particles. Dark matter particles have to interact with one another, to allow for elastic collisions and thermal equilibrium, although they escape from short-range interactions with particles of ordinary matter.

Let us consider a quasi infinite "cosmic wall". At a distance x from its central plane there exists a gravitational force that is oriented towards the wall and characterised by the acceleration g(x). Its value is the integral of $2\pi G(\rho+\rho')dx$, when ρ and ρ' are respectively the densities of dark matter and ordinary matter in sheets of thickness dx. Hydrostatic equilibrium is achieved when $dP = -\rho g dx$ or $d\rho/dx = -(mg/kT)\rho$. For a symmetric wall, we have to substitute

$$g(x) = 4\pi \int_o^x (\rho + \rho')\, dx$$

Assuming that the distribution of ordinary matter results from the distribution of dark matter, we set $\rho'(x) = \alpha\rho(x)$, where the constant $\alpha < 1$. Writing $\rho(x) = \rho_o y(x)$, so that $y(0) = 1$, we get the equation

$$\frac{dy}{dx} = -(2/\lambda^2)\, y \int_o^x y\, dx, \qquad \text{where} \qquad \lambda^2 = \frac{kT}{2\pi G\,(1+\alpha)\rho_o\, m}$$

Its solution is $y(x) = \text{sech}^2(x/\lambda)$ and the total amount of dark matter par unit surface is $2\lambda\rho_o$. This value is constant, when particles of dark matter can only be created during the primeval Big Bang. Cosmic walls had to contract during the ensuing free expansion of the universe, since λ is proportional to the diminishing temperature T.

Dark matter particles are still unidentified, but the primeval particle-creating period was such a high-energy event that many types of particles were created, even those which could not yet be produced by terrestrial accelerators. All particles decayed, to leave only quarks and leptons of ordinary matter, with a cosmological background of photons and neutrinos. *Narks and antinarks* of the first family could survive, however, if they were preserved from mutual annihilation inside adequate structures.

Nucleons or baryons result from the association of three quarks in different color states. When we represent the occupied color states by dots (figure 6a), we can say that mutual binding results from opposite shifts of two neighboring dots, with the emission and rapid absorption of a color-changing gluon. Six narks can also form a stable entity (figure 6b), when mutual annihilations are prevented by the presence of a neutrino or antineutrino. Since the resulting spin 1/2 particle is electrically neutral and analogous to a nucleon, we propose to call it a "neutralon". Particles of this type could still be present in interstellar and intergalactic space.

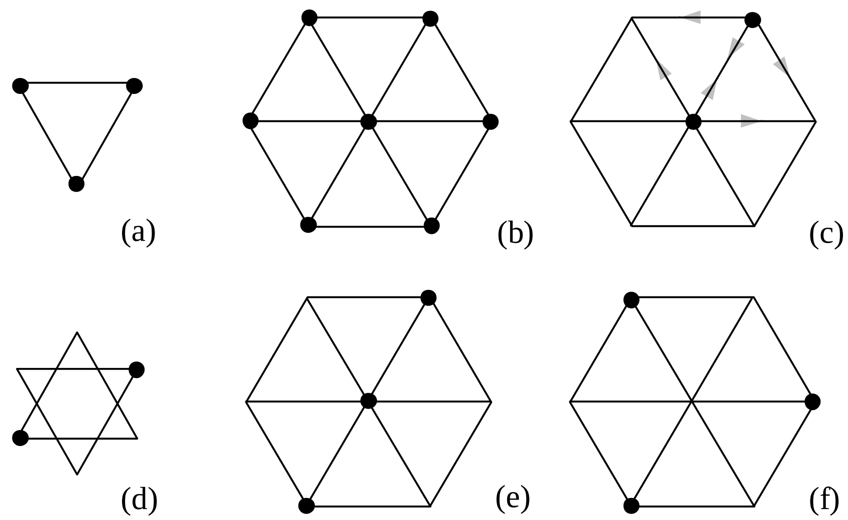

Fig. 6. Composite particles, where occupied states are represented by dots:
(a) three quarks, forming a baryon, (b) associated narks, forming a heavy neutralon,
(c) a bound nark pair, (d) a quark-antiquark pair, forming a meson,
(e and f) possible configurations of a light neutralon.

It is necessary, however, to show that the formation of neutralons was realizable in the primeval "soup" of elementary particles by means of a plausible sequence of steps. The fist one could bind two colored narks [or a colored nark and a color-neutral nark] to one another (figure 6.c). The resulting boson is colored, but similar to a meson (figure 6.d), where the binding of a quark and antiquark results from parallel, but opposite shifts of dots. The nark pair can capture the colored anti-nark (figure 6.e). This allows for an alternative configuration (figure 6.f) and for the constitution of a colorless fermion, because of all possible shifts of dots in opposite directions. Let's call it a "light neutralon". In its non-diagonal configuration, it can capture a color-less nark and then another light neutralon in the complementary non-diagonal configuration, or vice-versa. The intermediate state is metastable, but the end product is a stable “heavy neutralon” (figure 6.b).

Nucleons can be bound to one another to form nuclei or neutron stars by exchanging quark-antiquark pairs (figure 6.d). Neutralons could also be bound to one another by exchanging virtual nark-neutrino pairs (figure 6.c). The formation of larger lumps of dark matter would then lead to a distribution of masses m for "cold dark matter". Neutralons can collide with one another, but they don't interact with usual baryons, because of occupied fermion states and baryon color-neutrality. [However, it might be possible to fuse or to fission heavy neutralons, with an energy release, so that energy could be available in interstellar space.]

Is it possible to create nark-antinark pairs and neutralons by means of accelerators? We don't know, but some recent experiments [24] seem to indicate that "new physics" is not excluded. Anyway, for the time being, it is only important to see that space-time quantization yields a classification of elementary particles that accounts in a natural, unified way for a large number of individual experimental results and suggests other observational tests.

3.7. The Generalized Dirac Equation

It is remarkable that all elementary particles, bosons as well as fermions, can be viewed as *excitations of space and time,* since gravitational interactions were also attributed to properties of space and time in general relativity. The "metric" is always determined by possible results of measurement. The classification of elementary particles, generalizing the standard model, should thus be deducible from equation (5), governing the possible variations of ψ-functions on all space-time lattices, although the relative phases are not yet apparent in (5).

In that regard, we have to recall that Dirac could account already for *some internal degrees of freedom* of elementary particles by showing that the differential Gordon-Klein equation is equivalent to a set of first-order differential equations. They allow for spin up and spin down states, as well as particle and antiparticle states. Initially [25], when we tried to find out if $a \neq 0$ could account for other internal degrees of freedom of elementary particles, we knew only about the "normal lattice". We had thus to define a translation operator for the smallest possible step: $T_{x+d}\, f(x) = f(x+d)$, where $d = \pm a$. This imposed two derivation operators:

$$D_x^+ = (T_{x+a} - 1)/a \qquad \text{and} \qquad D_x^- = (1 - T_{x-a})/a \qquad \text{with} \qquad D_x^+ D_x^- = D_x^2$$

It was possible to obtain a generalized Dirac equation with D^+ and D^- operators, and it did account for new internal degrees of freedom, interpreted by means of "inserted lattices". Here, we introduced this concept and the resulting degrees of freedom in a much more direct way. We can thus define the symmetric operator

$$D_x = (T_{x+a/2} - T_{x-a/2})/a \tag{9}$$

When this is done for all space-time axes, we can immediately transpose Dirac's procedure. Equation (5) is equivalent to the *generalized Dirac equation*

$$\left(D_{ct} + \Sigma_j \alpha_j D_j + i\beta\mu\right)\Psi = 0 \tag{10}$$

where α_j and β are the usual Dirac matrices and j = x, y, z, while Ψ is a 4-component spinor. In the last term of (10), Ψ is specified on the normal lattice, privileged by the experimental set-up, while the D operators generate spinors on inserted lattices. In the *continuum approximation,* $D_x\Psi \to U_x\partial_x\Psi$ and $\mu \to \mu_o = m_oc/\hbar$. The usual Dirac equation is replaced by

$$\left(U_{ct}\partial_{ct} + \Sigma_j \alpha_j U_j \partial_j + i\beta\mu_o\right)\Psi = 0 \tag{11}$$

The U-factors could have been introduced long ago, by linearizing Einstein's energy-momentum relation, but the usual concept of derivatives in a space-time continuum did not suggest that $\partial_x = (U_x\partial_x)^2$, where $U_x = \pm1$. The existence of elementary particles is compatible with differential equations, but only as an approximation!

When a spin 1/2 particle carries an electric charge q, we can describe its interaction with an electromagnetic field in the usual continuum approximation by substitutions of the type $\partial_x \to \partial_x + i(q/\hbar c)A_x$. This should also be possible for (11). To prove this, we use Einstein's relation (2) and the idea that a charged particle behaves in an electromagnetic field, characterized by a scalar potential ϕ and a vector potential **A**, as if it were a free particle when the *local values* of E and p are redefined so that

$$(E + q\phi)^2 = (c\mathbf{p} + q\mathbf{A})^2 + (m_oc^2)^2 \tag{12}$$

The spinor Ψ is simply subjected to a *gauge transformation:* $\Psi \to \Psi' = \Psi e^{i\alpha f}$, where f is a function of space and time, chosen in such a way that the effects of the electromagnetic field are not apparent any more at a sufficiently small scale. In the generalized theory, the spinor Ψ' has to satisfy equation (10). To evaluate $D_x\Psi'$, we apply (9) to the product of two functions F and G:

$$D_x(F\cdot G) = (D_xF)\cdot(S_xG) + (D_xG)\cdot(S_xF) \quad \text{where} \quad S_x = (T_{x+a/2} + T_{x-a/2})/2$$

In the continuum approximation, $D_x\Psi = U_x\partial_x\Psi$ and $S_x\Psi = U_x\Psi$. When the function f is defined on all space-time lattices, so that the phases on the inserted lattices with respect to the normal lattice are defined by $V_x = \pm1$, we get $D_xe^{i\alpha f} = i\alpha V_x\partial_xf\, e^{i\alpha V_xf}$ in the continuum approximation, and $D_x\Psi' = U_x(\partial_x + i\alpha V_x\partial_xf)\Psi e^{i\alpha V_xf}$, while $S_xe^{i\alpha f} = e^{i\alpha V_xf}$. The U-factors and V-factors characterize respectively the fermion and boson field. Since photons are [0, 0, 0, 0] particles in u-space, all V-factors are equal to 1. It follows that for electromagnetic interactions $D_x\Psi' = U_x(\partial_x + i\alpha\partial_x f)\Psi e^{i\alpha f}$. We get (11), with the expected substitution when $\alpha = q/\hbar c$ and $A_x = \partial_x f$,... The theory is consistent.

Although fermion-states are characterized by (u_x, u_y, u_z, u_{ct}), it is sufficient to define the electric charge q = Qe by (8). The vector potential **A** and the scalar potential ϕ are subjected to the Lorentz condition, for instance, leaving only one vector field. Although the spatial u-quantum numbers can be different, they have an equal status for electromagnetic interactions, since all V-factors are identical. Lattice gauge theories associated gauge currents with "links" between neighboring lattice points [19], while we defined boson-fields as well as fermion-fields only at lattice-points where particles could be localized, in principle. Matrix elements for transition probabilities are thus given by the sum of products of functions that are defined at the same lattice-points. This allows for a smooth transition to the usual continuum theory.

3.8. The Velocity Operator

Pauli [7] showed that three different definitions of the average velocity v of a material particle yield identical results in *non-relativistic* quantum mechanics. He considered (i) the time derivative of the average position, (ii) the probability current density and (iii) the group velocity of a wave-packet. This proof of *internal consistency* is still valid when $a \neq 0$ and $c = \infty$, but it appeared [26] that the local value of v has to be defined by

$$v\psi = -i\hbar(T_{x+a} - T_{x-a})\psi/2am_o \tag{13}$$

This means that $v = p/m_o$, where the local value of p is defined by $-i\hbar\partial_x$ in the continuum approximation, but ∂_x is not replaced by (9). This is quite astonishing, at first sight, but it results from the fact that the velocity v characterizes states of motion. They are defined by variations of ψ on any particular space-time lattice of lattice-constant a. (13) implies that $v = (\hbar/m_o a)\sin(ka)$ when the momentum $p = \hbar k$. This is in agreement with (4), since by setting $s = \pi a/h$ and taking the square root of E^2 when $c \to \infty$, we get

$$E = m_o c^2 + \frac{\sin^2(ps)}{2m_o s^2} \qquad \text{and} \qquad v = \frac{dE}{dp} = \frac{\sin(ap/\hbar)}{m_o a/\hbar}$$

For a free particle that is moving along an arbitrary direction in the chosen inertial frame, we have to use the sum of similar kinetic energies for all momentum components p_x, p_y and p_z. When the particle carries an electric charge q and interacts with electromagnetic radiation, it is sufficient to replace p_j by $p_j + qA_j/c$, the scalar potential ϕ being equal to zero. Retaining only first order terms in 1/c, it is necessary to add q(**A.v**)/c to the free-particle energy E, as in the usual theory, but the components of the velocity vector **v** are given by (13).

4. Spacetime and the Universe

4.1. The Generalized Lorentz Transformation

The relativistic invariance of *c* is usually expressed by the Lorentz transformation for space-time coordinates. It is equivalent to a rotation of orthogonal (x, ict) and (x', ict') axes, when x and x' are measured along the direction of relative motion. When (x, ct) and (x', ct') are quantized, we can only get a superposition of particular lattice points The resulting *sub-group* of Lorentz transformations [9] is not sufficient since the choice of reference frames should be unrestricted, but we don't need such a deterministic correspondence for quantized observables. A probabilistic one is adequate.

It is usually expressed by a change of representation of ψ in terms of possible results of measurement. Any wave-packet can be defined on two different space-time lattices, with the same global shape of the probability distribution and the same calculated average position. It is more convenient, however, to use the physically equivalent energy-momentum representation, since these observables are not quantized in our (sufficiently large) universe. We get a deterministic correspondence between sharply defined values (E, p) in one inertial frame and sharply-defined values (E', p') in the other inertial frame. The amplitudes of corresponding Fourier components of ψ are identical, and the requirement of relativistic invariance of the energy-momentum relation (4) yields the *generalized* Lorentz transformation [11]

$$\begin{aligned} \sin(\pi a E'/ch) &= \gamma\left[\sin(\pi a E/ch) - \beta \sin(\pi a p/h)\right] \\ \sin(\pi a p'/h) &= \gamma\left[\sin(\pi a p/h) - \beta \sin(\pi a E/ch)\right] \end{aligned} \qquad (14)$$

The parameter β specifies the relative motion of the chosen inertial frames, while $\gamma^2 = 1/(1-\beta^2)$. To measure β, we have to use *a material reference object*. Being at rest in one frame (p' = 0), it has an energy E and a momentum p in the other frame, so that β is determined by the second relation (14). Since v = dE/dp, it follows from (4) that

$$\frac{v}{c} = \frac{\sin(ap/h)}{\sin(aE/ch)} \qquad \text{while} \qquad \beta = \frac{\sin(ap/2h)}{\sin(aE/2ch)}$$

In the continuum approximation, we get β = v/c. The mass of the reference object is then irrelevant, but this feature of special relativity is inconsistent with quantum mechanics. Brillouin [27] noted already that the (usual) Lorentz transformation requires that *the position and the velocity* of the origin of one frame can be measured with absolute precision with respect to another frame, although this is incompatible with Heisenberg's uncertainty relations. The effects of *c* and *h* don't fit together. Physical reference frames cannot simply be treated as mathematical coordinate systems, and we can't even claim that measurements should be performed with macroscopic equipments, since relative motions are possible for microscopic objects.

Brillouin's paradox indicates, once more, that the effects of *c* and *h* have not yet been combined in a satisfactory way. The field equation (3) of relativistic quantum mechanics contains *c* and *h*, but it is equivalent to the energy-momentum relation (2) where *h* does not appear. This results from the (implicit) assumption that $a = 0$. Space-time quantization yields a better blend: all relations (4), (5) and (14) contain *c*, *h* and *a*.

4.2. Superluminal Velocities, Inertia and Causality

For freely moving material objects, the group velocity v = dE/dp = v(p), where p is the average momentum in the chosen inertial frame. Considering only motions along a given direction, we define the magnitude of the *applied force* by F = dp/dt. The time variable t can be treated as if it were continuous, although *c* is finite, when the variation of p is sufficiently smooth at the scale of the quantum of time *a/c*. The acceleration

$$\frac{dv}{dt} = \frac{dv}{dp}\cdot\frac{dp}{dt} = \frac{F}{m} \qquad \text{where} \qquad \frac{1}{m} = \frac{dv}{dp} = \frac{d^2E}{dp^2}$$

The velocity dependent longitudinal inertial mass is determined by (4):

$$m(v) = \frac{\sin(aE/ch)}{(ac/h)\{\cos(ap/h) - (v/c)^2\cos(aE/ch)\}} \quad \rightarrow \quad \frac{E/c^2}{1-(v/c)^2}$$

in the continuum approximation, where $E = \underline{m}c^2$ and $p = \underline{m}v$, with $\underline{m} = m_o[1-(v/c)^2]^{-1/2}$. This yields the well-known value $m(v) = m_o[1-(v/c)^2]^{-3/2}$, when $a = 0$. To get the lowest order correction, we have to multiply this expression by $(1-\varepsilon^3/6)$, where $\varepsilon = \pi E/E_u \ll 1$. The usual approximation is very robust, but m(v) increases when $v \rightarrow c$. Beyond the light barrier, m(v) decreases for increasing velocities. We define the *height of the light barrier*, where v = c and $E = E_u - pc$, by $m(c) = -(E_u/2\pi c^2)\,\mathrm{tg}(\pi E/E_u)$. It is infinite for usual material bodies, since $E \approx E_u/2$ when $E_o \ll E_u$ and v = c, but m(c) decreases when $E_o \rightarrow E_u$. We can generalize special relativity for average motions, although E_u is finite.

It has often been stated that superluminal velocities violate the *principle of causality*. This results from the peculiar properties of hypothetical particles, endowed with an imaginary rest-mass in the *continuum approximation*. Since $(m_oc)^2$ is negative in (2), the E(p) curves are hyperbolas that cut the p axis. The velocity v = dE/dp is thus infinite when $E = 0$. It remains superluminal for increasing energies. When such a "tachyon" travels a distance $\Delta x = v\Delta t$ in one inertial frame, the (usual) Lorentz transformation implies that $c\Delta t' = \gamma(c\Delta t - \beta\Delta x) = \gamma(c-\beta v)\,\Delta t$ in an other inertial frame, with $\gamma^2(1-\beta^2) = 1$ and $\beta < 1$. The order of events will be reversed when $v > c/\beta$. The principle of causality can thus be violated [28].

If this could happen for superluminal velocities that are predicted by spacetime quantization, we would have to reject this theory. It applies to objects with a real rest-mass, but we can't die for some observers before we are born. This absurdity is prevented by the generalized Lorentz transformation (14), since it modifies the law for the addition of average velocities [11].

4.3. Faster than Light and the EPR Paradox

Einstein expressed already in 1927 a "deep concern over the extent to which *causal account in space and time* was abandoned" in wave-mechanics [30]. The EPR paradox [13] demonstrated this feature for two particles 1 and 2, emitted in opposite directions with a common ψ-function. It is then sufficient to perform a measurement on particle 1 to acquire instantaneously new knowledge about particle 2, however far it may be from particle 1. We can't explain these "spooky" actions at a distance. They should even be impossible, according to the special theory relativity.

Einstein [29] considered that quantum mechanics and relativity are "both correct in a certain sense", but "their *combination* has resisted all efforts up to now". Relativistic quantum mechanics accounts for the effects of c and h to allow for creation and annihilation processes, but this is not sufficient. How could we get a more general theory? Einstein warned: "a theory can be tested by experience, but there is no way from experience to the setting up of a theory". He suggested that non-linear equations might be necessary, as in general relativity. They would modify the concept of space and time. However, Einstein indicated the probable source of the fundamental difficulties: quantum mechanics uses "basic concepts, which on the whole have been taken over from classical mechanics".

When Aspect's experiments [31] confirmed the correctness of quantum-mechanical predictions for two particle systems, d'Espagnat [32] noted immediately that this seems to require *superluminal* velocities. Penrose [33] stated also that the EPR paradox calls for "an influence that must travel faster than light", although this is "in definite conflict with the spirit of relativity". He added: "it would be surprising if quantum mechanics would not undergo some fundamental change in the future" and this could imply "some very radical new ideas about the nature of spacetime geometry".

We generalized quantum mechanics, by defining ψ-functions on space-time lattices, but much to our surprise, this changed also an important result of the theory of relativity:

superluminal velocities are not excluded anymore! The behaviour of real (observable) particles is unaffected at normal energies, but *virtual particles* can have extremely high energies, since they are not restricted by Heisenberg's uncertainty relations. The divergence difficulties resulted from the fact that they could even be infinite when $a = 0$. The ψ-function of two entangled particles accounts for *quantum mechanical potentialities*, including communications at superluminal velocities through exchanges of virtual particles. Since their energies are limited by $E_u = hc/2a$, the divergence difficulties are removed as well as the EPR paradox, when $a \neq 0$.

It should be noted that the concept of ψ-function imposed already a *holistic* view of reality. These functions were defined in all space at any given instant, and they could be instantaneously modified everywhere by a local measurement. This was acceptable for a single particle, since ψ is only *an epistemological tool*. It specifies the knowledge we have about the particle. This knowledge is instantaneously modified by a particular measurement. However, the ψ-function of two or more particles led to a very serious problem, since it implied the existence of a connection between distinct *elements of reality*. To speak about "non locality" or quantum-mechanical "inseparability" is not sufficient. We have to explain the mechanism. Pauli's exclusion principle for two or more spin 1/2 particles raised an identical problem. The existence of superluminal velocities for virtual particles was already foreshadowed by the usual theory, but we couldn't understand this feature as long as we believed in a space-time continuum.

The quantum-mechanical entanglement of a two-particle system can be exploited to achieve teletransportation of a particle state over large distances [34]. It has also been experimentally verified [35] that tunneling allows for a transmission that is "faster than light", according to the time required to cross the barrier. Transmitted laser light can even carry musical information at 4.5 times *c*, for instance. Free photons have to travel at the velocity *c*, of course, but the transmission of evanescent electromagnetic waves implies interactions and the uncertainty relations. This shows once more that superluminal velocities appeared before we knew that they are possible.

4.4. The Definition of Inertial Frames

The analysis of motions involves space and time and therefore *the whole universe*. In Aristotelian mechanics, it was assumed that the universe is limited by the sphere of stars and that its center is in a state of "absolute rest". It was then possible to make an absolute distinction between motion and rest for all bodies by referring to this *privileged point*, assumed to coincide with the center of the Earth. Motions seemed to be some kind of life. Galilei recognized, however, that velocities have no absolute meaning, since they can be added to one another. This is equivalent to a change of reference frame, and there is no privileged frame among all those which are unaccelerated with respect to the Earth.

Newton integrated these ideas in his new philosophy of Nature. It was based on the concept of forces. They can be of various types, but all forces are measurable by means of the accelerations they produce by acting on a body of given mass. It is necessary, however, to use an "inertial reference frame". It can be freely chosen among all those who are unaccelerated relative to one another, but *their ensemble is privileged,* since they are the only frames where the acceleration is zero when the applied force is zero. Newton justified his "principle of inertia" by means of a very ingenious conceptual adjustment. Instead of postulating that there exists *only one* point in the whole universe that is in a state of absolute rest, he assumed that there exists *everywhere* a point that is motionless. The ensemble of these points defines "absolute space" and inertial frames are those frames which are unaccelerated with respect to absolute space.

It was not more difficult to accept the existence of "absolute time", flowing always and everywhere in the same way. Space and time were actually considered as physical realities, belonging to creation, like matter and forces. Einstein abandoned the concept of absolute space and time, to allow for the constancy of *c*, but he recognized that this raises a fundamental problem [29]: *"How does it come about that inertial systems are physically distinguished above all other coordinate systems?"* Mach's principle required that we should use observable reference objects, like "fixed stars" or a large portion of the universe, but why should local inertia be related to distant objects?

The generalized energy-momentum relation (4) solves this problem, since it yields a new definition of inertial frames. They are not only those frames where the principle of inertia is valid (E and p are constants for freely moving particles), but they are also related to the whole universe (a system of rest-energy $E_o = E_u$ would be unable to move). The old intuition that something has to be at "absolute rest" was correct, but it is not a single point, nor absolute space. We consider the limit, where the inertial frame is bound to a very small part of the universe, while all the rest is lumped together. Since the total convertible energy content of our universe is finite, the system of highest possible rest-energy E_u has to be at rest, but this state (p = 0) is defined in the quantum-mechanical sense. The position is then undetermined.

4.5. Cosmology

The total rest-mass M of our universe is defined by $E_u = Mc^2$ in a particular inertial frame. The cosmological principle and the finite value of M imply that this mass should be uniformly distributed on the surface of a hypersphere of radius R, when the average mass density ρ is defined at a sufficiently large scale. We can imagine a sphere of radius r in a three-dimensional (x_1, x_2, x_3) space and a circle of radius R on a (r, x_4) plane. The surface of the sphere of radius r is $4\pi r^2$. Using polar coordinates (χ, R) for the circle, we get $r = R \sin\chi$, while the length of the arc $R\chi$ defines the distance between an arbitrarily chosen reference point O and an other point on the surface of the hypersphere. The volume up to $R\chi$ is

$$V_\chi = \int_o^\chi (4\pi R^2 \sin^2\chi)\, R\, d\chi \;=\; 2\pi R^3(\chi - \sin\chi \cos\chi)$$

Since the angle χ can increase from 0 to π, the total volume is finite [36]. The total convertible energy content of our universe

$$E_u = hc/2a = Mc^2 = 2\pi^2 \rho R^3 c^2 \qquad (15)$$

The energy E_u and the quantum of length *a* are universal constants, since M is constant, although ρ and R are time dependent. This applies to any local *inertial frame,* attached to the surface of the freely expanding hypersphere.

What happens to gravitational interactions? In general relativity, their effects are treated by considering modifications of the space-time metric. At cosmological scales, there is a time dependent scale factor R(t) and there should thus exist a time dependent quantum of length a(t). How can this be? In a certain sense, gravitational effects are predominant at a universal scale. The mass M, concentrated on the surface of a hypersphere of radius R, contains a certain amount of gravitational energy. To evaluate this energy, we consider the mass within a sphere of radius $R\chi$. It is $m_\chi = \rho V_\chi$, while the mass in the surrounding spherical shell of thickness $Rd\chi$ is $dm_\chi = \rho 4\pi R^3 \sin^2\chi d\chi$. The total gravitational energy

$$E_g = G\int_o^\pi \frac{m_\chi\, dm_\chi}{R\chi} \;=\; G\rho^2\, 8\pi^2 R^5 \int_o^\pi \left(\sin^2\chi - \frac{\sin 2\chi}{4\chi} + \frac{\sin 4\chi}{8\chi} \right) d\chi$$

The last integral $I = [\pi - Si(2\pi) + Si(4\pi)]/2$, where Si(x) is the sine-integral function. Thus, $I = (\pi/2)\gamma$, where $\gamma = 1 + 1/4\pi^2 + \ldots$ Since the total mass M is defined by (15), we get $E_g = \gamma GM^2/\pi R$, where G and M are constants. Since gravitational forces are always *attractive,* even for antiparticles, we get a negative gravitational potential energy $-E_g$, but it is reasonable to assume that the universe arose from a vacuum fluctuation and that its total energy remained always equal to zero. The energy E_g that has to be added to the gravitational potential to get E = 0 can be considered as another type of available energy in our universe: $E_u(t) = hc/2a = \gamma GM^2/\pi R$. The smallest measurable distance a(t) is now defined in a different way. It is proportional to R(t). The singularity $a = 0$ is avoided, when the Big Bang started by means of a primeval *particle creating period.*

Such a model was introduced by Gunzig, Brout, Englert, Prigogine et al. [37] and then elaborated by means of the concept of an inflationary universe.

The gravitational potential appeared simultaneously with matter and antimatter, but it is essential to note that the definition of the energy depends on the chosen reference frame. Why could gravitational effects be neglected in (15)? Why is the quantum of length a universal constant for inertial frames? Any relatively small mass m will be subjected to local gravitational forces in such a frame, but large-scale effects cancel out by symmetry. It is necessary, however, to prevent a gravitational collapse of the universe by means of kinematical effects. The free expansion of the hypersphere is equivalent to relative motions of all material objects on the surface of this sphere. The distance $r = \chi R$ between two points increases with a velocity $v = dr/dt = Hr$, where Hubble's constant H is the logarithmic derivative of R(t). The chosen inertial frame is attached to a particular point O on the surface of the hypersphere. A mass m that is situated at a distance r from O will thus have a rest-energy mc^2, increased by an (apparent) kinetic energy $mv^2/2$ and a gravitational potential energy GM'm/r, where $M' = 3\pi\rho r^3/3$ is the total mass inside the sphere of radius r. The resulting energy

$$E_m = mc^2 + mv^2/2 - GM'm/r. \tag{16}$$

Since the choice of O is arbitrary, the two last terms have to cancel one another. This yields $v^2 = 2GM'/r$ and Hubble's law, with $H = 8\pi G\rho/3 = 1/T$. Although the universe is closed, it seems to be flat. By measuring H, we can thus determine the critical mass density $\rho \approx 10^{-27}$ kg/m^3 and the age $T \approx 10^{10}$ years. Since (16) is reduced to $E_m = mc^2$, the total convertible energy of our universe in an inertial frame is the sum of all rest-energies, as required by (15). When $R \approx cT$, we get $a \approx 4\pi^2\rho hc^7T^3 \approx 10^{-94}$ m.

This value should only be considered as an estimation of *a,* but it shows that the quantum of length is much smaller than Planck's length $l_o \approx 10^{-35}$ m. This value results from a combination of *c, h* and *G.* It is also a universal constant, but it defines only the scale of wormholes and foam-like structures in the *continuum approximation of quantum-gravity.* It is a natural unit of length for a particular problem, like the Bohr radius a_o in atomic physics, and not the smallest measurable distance.

Conclusions

The essential result is that *the continuum assumption is not a logical necessity,* since it was possible to construct a consistent theory where the smallest measurable distance $a \neq 0$. This is analogous to the construction of Non-Euclidean Geometries. We tried very hard and patiently [10] to discover at least one internal contradiction, but we found none. On the contrary, it appeared that the existence of a finite quantum of length eliminated inconsistencies that subsisted in the usual continuum theories. Spacetime quantization could even account for a large set of highly remarkable experimental results in elementary particle physics. Moreover, it provides a natural extension of the standard model, suggesting further tests at accelerators or in the cosmic laboratory.

Steven Weinberg [38] discussed the search for new fundamental laws of nature in the present context. He insisted: "we certainly do not have a final theory yet", since "the problem of infinities is still with us" and since the standard model of elementary particle physics belongs to effective field theories that "are only low-energy approximations to a very different theory". There are too many empirical parameters, while "the aim of physics at its most fundamental level is not just to describe the world but to explain why it is the way it is". We know that "quantum mechanics by itself is not a complete physical theory. It tells us nothing about the particles and forces that may exist". However, it appeared also that "any *small change* in quantum mechanics would lead to logical absurdities".

A more profound conceptual change, touching the foundations of physics, seems to be necessary. Spacetime quantization provides it by going one step further along the direction that was pointed out already by the development of relativity and quantum mechanics. *Since Nature can impose restrictions on our measurements, there could be three of them instead of two!* The importance of measurements for our understanding of reality is then the unifying principle of 20th century physics. It is not only possible and less arbitrary to introduce a yet unknown quantum of length *"a"*. This yields also a more coherent picture of reality, since all three restrictions involve space and time, as well as inertial reference frames.

The universal constants *c, h* and *a* allow us, moreover, to measure and thus to define lengths, times and masses (or energies) in an unambiguous operational way anywhere in our universe. This was necessary to transform mechanics into a general science of motions, without privileging a special type of particles or interactions. The profound unity of our universe appeared already through Newton's principle of inertia, since it applied always and everywhere, for all material bodies and all forces. This coherence stems from *the common origin of everything,* and it is now emphasized by the fact that *c, h* and *a* are related to one another by the expression of the total convertible energy content of our universe: $E_u = hc/2a$.

We stressed the fact that ψ-functions have a holistic meaning, since they can be defined in the whole universe at any particular instant. The ψ-function of several particles implied an apparently magic connection, but now we see that it results from the possible exchange of virtual particles at superluminal velocities when $a \neq 0$.

Although spacetime quantization seems to be *logically possible and physically useful,* it requires a change of deeply rooted habits of thought. It may seem too simple and one can consider that its achievements are purely fortuitous. However, it raises at least new questions concerning the foundations of physics, by showing that *the present theory could be an approximation of a more general one*. The proposed classification of elementary particles could open new roads for theoretical and experimental research.

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